COMPUTER-AIDED ANALYSIS OF MAGNETIC FIELD IN ELECTROMAGNETIC SYSTEMS

- BIEM FOR MAGNETIC FIELD ANALYSIS -

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1. INTRODUCTION.

Modern development of computer applications such as CAD requires high accuracy of the magnetic field analysis in electromagnetic systems. Numerical methods like Boundary element method (BEM) or Finite element method (FEM) may be applied for a wide range of electromagnetic systems. These methods allow the field to be computed with high accuracy in an arbitrary point of area, its topology to be analysed, and its integral characteristics to be determined (e.g. inductance, force, etc.).

The boundary integral equation method may be used to solve a lot of electromagnetic problems. There are many advantages for this method of analysis. Its largest advantage is the reduced dimensionality of the mesh, i.e., the three- and two-dimentional problems require the two- and one-dimentional mesh systems, respectively. Furthermore, other advantages include the ability to model unbounded problems easily and the reduced number of equations.

Magnetic field of many electromagnetic systems can be considered 2D-plane or axisymmetrical (i.e. electromagnetics, reactors, etc.). The results from 2D-magnetic field calculations can be used to determine the electromagnetic parameters and characteristics of various devices. In this paper, constrained integral equations are derived for solution of two-dimensional magnetostatic problems. In order to solve this governing equations, numerical method is employed and its computer code is developed. Thus, we demonstrate the effectiveness of the computer aided analysis and design of the electromagnetic systems.

2. BOUNDARY INTEGRAL EQUATIONS

The electromagnetic fields of different devices are most completely described by Maxwell's equations [1]. Typical magnetic problems involve ferromagnetic bodies n with internal and external permeabilities $\mu_m^{(i)}$ and $\mu_m^{(e)}$ $(m=1,\cdots,n)$ correspondingly, which are surrounded with contours Γ_m. The current regions are l with current I_p $(p=1,\cdots,l)$. The permeable medium is assumed linear and homogeneous (see Fig. 1). For many electromagnetic systems, it can be assumed that all kinds of active losses in the system are neglected and two dimensional magnetic field consideration is possible. A vector potential formulation is well suited for two dimensional problems.

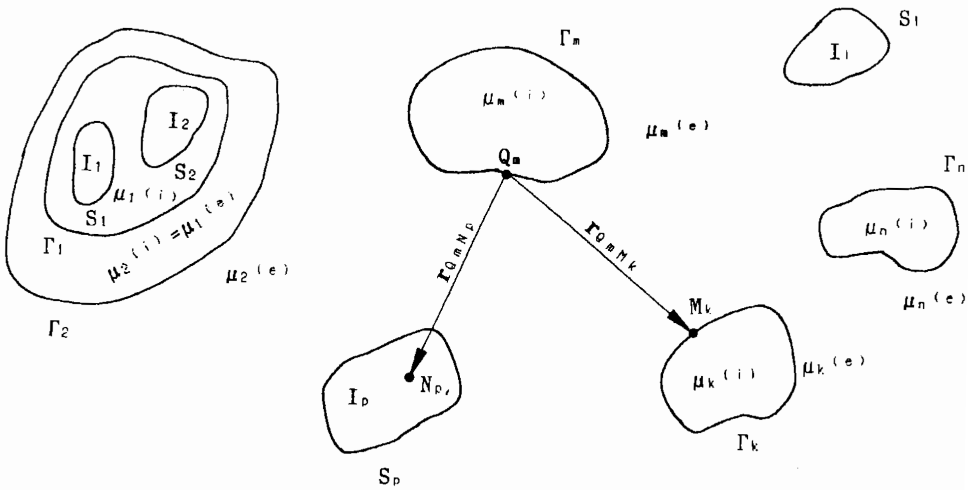


Fig.1 Two dimensional model of electromagnetic system.

The introduction of a vector potential **A** with **B**=rot **A** leads to the Poisson equation

$$\nabla^2 \mathbf{A} = -\mu \mathbf{J}_e, \tag{1}$$

K(k) and E(k) are the complete elliptic integrals of the first and second kind with module k defined by the expression

$$k^2 = \frac{4\rho_{Qm}\rho_{Mk}}{(\rho_{Qm}+\rho_{Mk})^2+(z_{Qm}-z_{Mk})^2} \quad ;$$

e_ρ , e_z - the unit vectors along the axes ρ and z;

s_{Qm} - the tangential vector to contour Γ_m in point Q_m.

The right side $F(Q_m)$ of SIE (5) is defined by the expression

$$F(Q_m) = \frac{\lambda_m}{\pi} \sum_{p=1}^{1} \iint_{(S_p)} \frac{j(N_p)}{\sqrt{(\rho_{Qm}+\rho_{Np})^2+(z_{Qm}-z_{Np})^2}} \quad (7)$$

$$\Big\{ \frac{(z_{Qm}-z_{Np})\cos(s_Q, e_p)}{\rho_{Qm}} \Big[K(k) - \frac{\rho_{Qm}^2+\rho_{Np}^2+(z_{Qm}-z_{Np})^2}{(\rho_{Qm}-\rho_{Np})^2+(z_{Qm}-z_{Np})^2} E(k)\Big] -$$

$$-\cos(s_{Qm}, e_z)\Big[K(k) + \frac{\rho_{Np}^2-\rho_{Qm}^2-(z_{Qm}-z_{Np})^2}{(\rho_{Qm}-\rho_{Np})^2+(z_{Qm}-z_{Np})^2} E(k)\Big] \Big\} dS_p ,$$

where N_p is the point from the current region (S_p);

$j(N_p)$ - the exciting current density of the region (S_p).

After solving of the SIE and determining the densities of secondary sources σ the vector potential is defined by the expression

$$A(Q) = \frac{\mu_0}{2\pi} \sum_{p=1}^{1} \iint_{(S_p)} j(N_p)\sqrt{(\rho_{Np}/\rho_Q)}\, f(k) dS_p +$$

$$+ \frac{\mu_0}{2\pi} \sum_{k=1}^{n} \int_{(\Gamma_k)} \sigma(M_k)\sqrt{(\rho_{Mk}/\rho_Q)}\, f(k) dl_k + C_A , \quad (8)$$

where $f(k) = (\frac{2}{k} - k)K(k) - \frac{2}{k}E(k);$

C_A - the arbitrary constant.

By numerical integration various integral parameters can be determined, on e.g. magnetic induction, fluxes, inductances, force etc.

Equations in SIE (3) and (5) are Fredholm integral equations of second kind. In many practical cases magnetic permeability $\mu \gtrsim (10^2 - 10^3)\mu_0$ and $\lambda \simeq 1$. Then the solution of integral equation is numerically quite sensitive and small numerical errors of the calculation of the right side may compromise the solution. In SIE (3) the term $\nu_{mk}(\pi/L_m)$ is included to improve the properties of the integral equations, when parameter λ is equal or near to characteristic value $\lambda \simeq 1$.

The term $\frac{\lambda_m}{L_m} \sum_{k=1}^{n} v_{mk} \int_{(\Gamma_k)} \sigma_k \, d l_k$ must be added to the right side of SIE when the contour includes current regions and sum of their currents $\Sigma I_p \neq 0$. In such cases this term determines the value of the right side.

3. NUMERICAL APPLICATION.

The systems of integral equations (1) and (5) are numerically solved by the mechanical quadrature method. A system of linear algebraic equations (SLAE) of high order is formed, which in a matrix vector form is as follows

$$[C] \cdot [\sigma] = [F], \tag{9}$$

where [C] is the coefficient matrix of SLAE;

[σ] - the vector of the unknown densities of secondary sources σ;

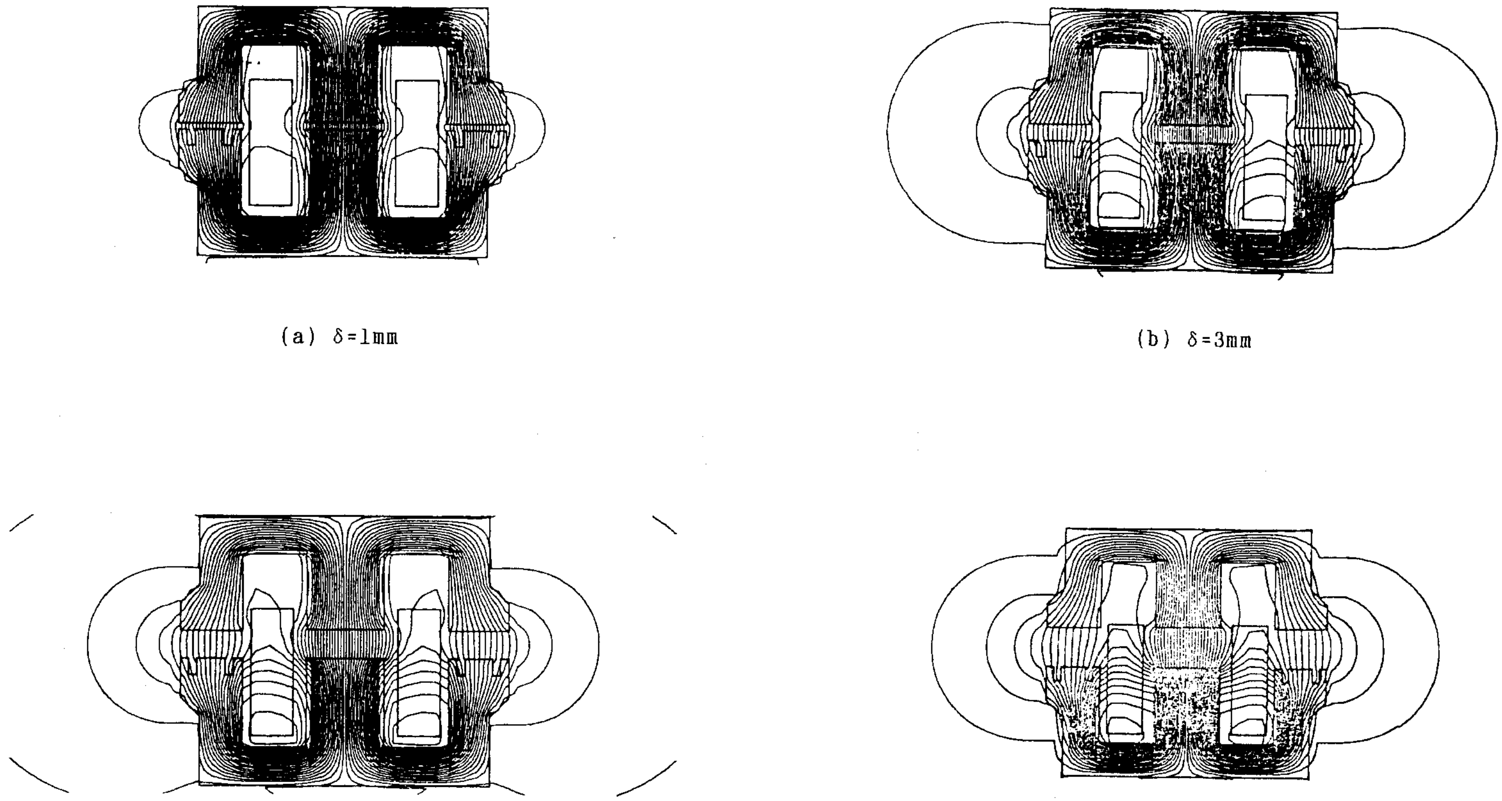
[F] - the vector of the right side of SLAE.

The cosine or regular distribution of boundary elements for each contour line may be applied. Analytical integration formulas can be used to derive the coefficient matrix [C] and boundary elements with constant or linear interpolation. When computing the right side of SLAE it is advantageous to use the analytical solution of the integral

$$J = \iint_{(S_p)} j(N_p) \left[\frac{\cos(\mathbf{r}_{Q_m N_p}, \mathbf{n}_{Q_m})}{r_{Q_m N_p}} \right] d S_p \;, \tag{10}$$

for a rectangular section (S_p) of the current region.

This leads to more accurate results not requiring the long CPU time [7]. The matrix of SLAE is asymmetrical, completed and of high order. For the solution the iterative method of Zeidel, the direct method of Gauss or the direct method using LU decomposition can be used. However, it is preferable to employ the iterative method of Zeidel in combination with direct or iterative methods. The initial approximation is obtained by preliminarily forming and solving a SLAE of low order. In the other words, the low-order solution is employed to form an initial approximate



(a) δ=1mm (b) δ=3mm

(c) δ=5mm (d) δ=8mm

Fig.2 Magnetic field distributions.

solution of the next higher order solution. This makes it possible to short the computational time for the solution of SLAE of high order. After computing the densities σ the vector potential **A**, the induction **B** in an arbitrary point of the magnetic field are determined by numerical integrating expressions. The boundary and surface integrals can be calculated by Gauss quadrature formulas. A computer program has been developed for the realization of BIEM and the visualization of magnetic field topology [7].

4. IMPLEMENTATION.

Magnetic field line patterns are plotted in Fig.2(a)-(d) for various air gaps of the electromagnet with double-E-formed core by neglecting the screen winding influence to illustrate the program performance. The system in consideration works in a mode of specified supply voltage for the winding. By applying BIEM the values of winding current and core permeability are iteratively established. Using the computer program, the magnetic fluxes, inductance, inductions, electromagnetic forces are possible to calculate. A lot of investigations are carried out for different constructions and size electromagnetics and reactors, working in different modes and conditions. The results obtained from calculations show a good agreement with experimental ones [7].

5. CONCLUSIONS.

The boundary integral equation method is a powerful tool for the analysis of electromagnetic problems with homogeneous media. Using this method, a lot of different electromagnetic systems are investigated. The developed computer program using BIEM allows the analysis of magnetic fields in electromagnetic systems to be carried out for different conditions and modes. Thus, this method is especially useful for the design and investigation of such systems.

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