Inverse Approach For Shape Design Of Magnetic Core

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***Abstract*—The paper proposes an inverse approach for the identification of magnetic material or for the shape design of a magnetic core. The magnetic material may be replaced with simple layer distribution of imaginary currents on its boundary. Thus, the inverse identification or magnetic core design problem can be reduced to source searching for the simple layer distribution of imaginary currents. The pattern matching figure is employed for the source position searching. The results obtained reveal that the approach is quite effective for solving such problems.**

I. Introduction

When dealing with inverse problems of electromagnetics (optimal field synthesis, optimal design, identification problems, etc.), a shape and location determination of a magnetic core is one of the central questions. This topic has recently become a subject of interest of several papers [1], [2]. This is an important part in the synthesis of electromagnetic devices from a given magnetic field it produced, as well as in nondestructive testing to identify the magnetic material in an unacceptable region using only the information from the surfaces.

In this paper, we propose an inverse approach to design the shape of a magnetic core in order to obtain given magnetic field distributions. Also, the same approach is applied to define the configuration and position of magnetic material in a region from a given magnetic field it produced. The proposed approach is based on the possibility to replace the magnetic material with simple layer distribution of imaginary currents on its boundary. Then, the magnetic field is considered as excited from simultaneously action of the coil currents and of the simple layer distribution of imaginary currents. Thus, the inverse problem for the shape and location identification of magnetic material or for the design of the magnetic core is reduced to the inverse source problem of the simple layer distribution of imaginary currents. Previously, in order to obtain the desired magnetic field pattern, the Sampled Pattern Matching method in [3], [4] used the pattern figure γ. In the proposed approach, the same relation γ is applied as a criterion to find the solution pattern.

To demonstrate the effectiveness of the proposed inverse approach, the results of some test examples for the identification of magnetic material and for the pole design of a magnetic core are presented. The analysis of these results reveals that the inverse approach is a useful tool for solving such problems.

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II. Inverse Problem Methodology

A. Governing equations

Most electromagnetic devices (e.g. transformers, electromagnets, reactors, etc.) are composed of magnetic cores with permeability μ and exciting coils with current density $\mathbf{J}$. The electromagnetic fields of such devices are described by following equation, assuming the Coulomb gauge $\nabla.\mathbf{A}=0$

$$\nabla^2\mathbf{A} = -\mu\mathbf{J}, \tag{1}$$

where $\mathbf{A}$ is the magnetic vector potential. Imposing a homogeneous open boundary condition, we have

$$\mathbf{A} = \mu\int_V \frac{\mathbf{J}}{4\pi\mathbf{r}}dV, \tag{2}$$

where $\mathbf{r}$ is the radius-vector between the points of the vector potential $\mathbf{A}$ and of the current integration.

The magnetic flux density is obtained from the vector potential $\mathbf{A}$

$$\mathbf{B} = \nabla\times\mathbf{A} = \nabla\times\mu\int_V \frac{\mathbf{J}}{4\pi\mathbf{r}}dV. \tag{3}$$

The existence of magnetic cores makes the media piece-wise homogeneous. This quasi inhomogeneity can be suspended by introducing a simple layer distribution of imaginary currents with density σ on the boundaries between media with different magnetic properties (boundary of the magnetic cores) and after that removing the magnetic cores. Thus, the magnetic core is replaced with a simple layer distribution of imaginary currents with density σ on its boundaries. The magnetic field at arbitrary point of the space is determined as a field in homogeneous media (vacuum), excited from the currents of the coils and simple layer distribution of imaginary currents.

Let us consider the electromagnetic device constructed from

p exciting coils and q parts of magnetic material with constant magnetic permeability μ_q in their volumes. Solving magnetostatic problems where two dimensional electromagnetic field consideration is acceptable and replacing the magnetic cores with a simple layer distribution of imaginary currents with density σ, the vector potential A and the magnetic flux density components B_x and B_y are expressed by

$$A(Q) = \frac{\mu_0}{2\pi}\sum_{i=1}^{2p}\int_{S_i} j(N)\ln\frac{1}{r_{QN}}ds + \frac{\mu_0}{2\pi}\sum_{j=1}^{q}\int_{L_j}\sigma(M)\ln\frac{1}{r_{QM}}dl + c_A \quad (4)$$

$$B_x(Q) = \frac{\mu_0}{2\pi}\sum_{i=1}^{2p}\int_{S_i} j(N)\frac{\cos(\mathbf{r}_{QN},\mathbf{e}_y)}{r_{QN}}ds + \frac{\mu_0}{2\pi}\sum_{j=1}^{q}\int_{L_j}\sigma(M)\frac{\cos(\mathbf{r}_{QM},\mathbf{e}_y)}{r_{QM}}dl \quad (5)$$

$$B_y(Q) = -\frac{\mu_0}{2\pi}\sum_{i=1}^{2p}\int_{S_i} j(N)\frac{\cos(\mathbf{r}_{QN},\mathbf{e}_x)}{r_{QN}}ds - \frac{\mu_0}{2\pi}\sum_{j=1}^{q}\int_{L_j}\sigma(M)\frac{\cos(\mathbf{r}_{QM},\mathbf{e}_x)}{r_{QM}}dl \quad (6)$$

where c_A, e_x and e_y are the arbitrary constant and unit vectors in x- and y-directions, respectively; M, N - the points of current integration from cross-section S_i of the i-coil and boundary L_j of the j-magnetic material, respectively; Q - the observation point; r_{QM}, r_{QN} - the distance between points Q and M, N, respectively.

The magnetic material has to be with constant magnetic permeability in its volume. If the magnetic material is non-linear then an average value of magnetic permeability is used.

B. System Equations of the Inverse Problem

If $\mathbf{C}$, $\mathbf{D}$ and $\mathbf{Y}$ are the n by m matrices and n-th order column vector of the vector potential $\mathbf{A}$ or the magnetic flux density $\mathbf{B}$, then (4) - (6) can be presented by the system equations

$$\mathbf{CJ} + \mathbf{D}\sigma = \mathbf{Y}, \quad (7)$$

where $\mathbf{J}$ and σ are the exciting current density and simple layer distribution of imaginary current vectors, respectively. Further, the vector Y is rewritten by

$$\mathbf{Y} = \mathbf{Y}_\mathbf{J} + \mathbf{Y}_\sigma. \quad (8)$$

The component $\mathbf{Y}_\mathbf{J} = \mathbf{CJ}$ is the externally impressed field source vector. It is caused by the exciting currents of the coils. The component $\mathbf{Y}_\sigma = \mathbf{D}\sigma$ is caused by the simple layer distribution of imaginary currents. If the location and currents of the exciting coils are known then

$$\mathbf{D}\sigma = \mathbf{Y} - \mathbf{Y}_\mathbf{J} = \mathbf{Y}_\sigma. \quad (9)$$

Equation (9) reveals that the shape and position identification or design problem of the magnetic core can be reduced to the source position searching problem of the simple layer distribution of imaginary currents from the magnetic flux density or potential distributions obtained by local measurements.

C. Unique Solution Pattern

In region Ω we search for the magnetic core shape and position in order to obtain the given magnetic field distribution around this region. The external impressed field of the exciting coils is known. The media in the search region Ω is homogeneous and isotropic. The measurements of the magnetic field (vector potential or magnetic flux density) are around the target region at local measurement points. The number of equations n corresponding to the number of measurements is much less than the number of unknowns m ($n < m$) of the imaginary current densities σ. Because of that it is difficult to obtain a unique solution of (9).

Equation (9) can be presented by

$$\mathbf{Y}_\sigma = \sum_{j=1}^{m}\sigma_j\mathbf{d}_j, \quad (10)$$

If the vector $\mathbf{Y}_\sigma$ corresponds to the vector potential $\mathbf{A}$, then the elements of the pattern vector $\mathbf{d}_j$ are

$$d_{ij} = \frac{\mu_0}{2\pi}\left[\ln\frac{1}{r_{Q_jM_{ij}}}\right], \quad i = 1,\ldots,n. \quad (11)$$

If the vector $\mathbf{Y}_\sigma$ corresponds to the components B_x or B_y, then the elements of the pattern vector $\mathbf{d}_j$ are

$$d_{ij} = \frac{\mu_0}{2\pi}\left[\frac{\cos(\mathbf{r}_{Q_jM_{ij}},\mathbf{e}_y)}{r_{Q_jM_{ij}}}\right]$$

or

$$d_{ij} = \frac{\mu_0}{2\pi}\left[\frac{\cos(\mathbf{r}_{Q_j M_{ij}}, \mathbf{e}_x)}{r_{Q_j M_{ij}}}\right], \quad i=1,\dots,n, \tag{12}$$

respectively.

On the boundary of the region Ω we search for the dominant positions of the simple layer distribution of imaginary currents with density σ. As a criterion the pattern figure

$$\gamma_j = \frac{\langle \mathbf{Y}_\sigma, \mathbf{d}_j \rangle}{\|\mathbf{Y}_\sigma\| \|\mathbf{d}_j\|}, \qquad j = 1,\dots, m. \tag{13}$$

is used.

This relation defines the angle θ between vectors $\mathbf{Y}_\sigma$ and $\mathbf{d}_j$ because

$$\cos\theta_{\mathbf{Y}_\sigma \mathbf{d}_j} = \frac{\langle \mathbf{Y}_\sigma, \mathbf{d}_j \rangle}{\|\mathbf{Y}_\sigma\| \|\mathbf{d}_j\|}. \tag{14}$$

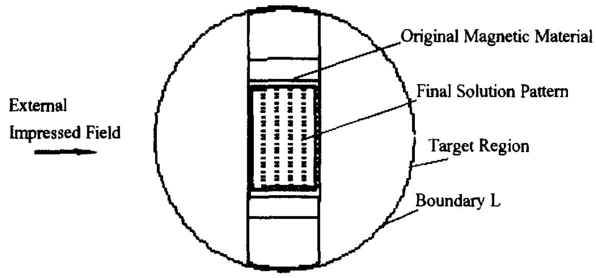
According to the Schwarz inequality the relation γ_j is between -1 and 1. The maximum value of γ_j ($\gamma_j = 1$) shows that the direction of the two vectors $\mathbf{Y}_\sigma$ and $\mathbf{d}_j$ coincides. This property of the pattern figure γ (13) can be used as a criterion to estimate the positions j of the simple layer distribution of imaginary currents on the boundary L. High values of γ correspond to the dominant positions of the simple layer distribution of imaginary currents. From all positions j on the boundary L we choose m_k of them $(m_k < m)$ where the pattern figure γ has a maximum value. These m_k positions are consequently connected creating a new boundary L. Using pattern figure γ (13) we estimate the positions of the simple layer distribution of imaginary currents on the this new boundary L and choose m_k of them creating new boundary L. Similar procedures continue until minimum value of the γ reaches a maximum value (min$\gamma \approx 1$). The last boundary determines the shape and location of the magnetic core.

The approach proposed above gives the unique solution pattern of the shape and location of magnetic core.

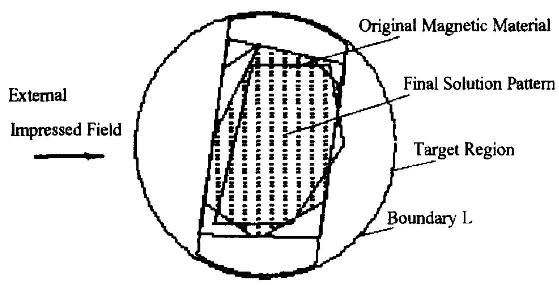
III. Examples

Some examples for magnetic material identification and for pole design of magnetic core demonstrate the usefulness of our inverse approach.

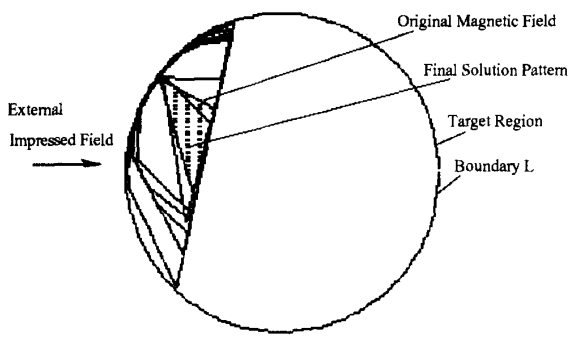
The results obtained solving the magnetic material shape and location identification problems are shown in Fig.1(a-c).



(a)



(b)

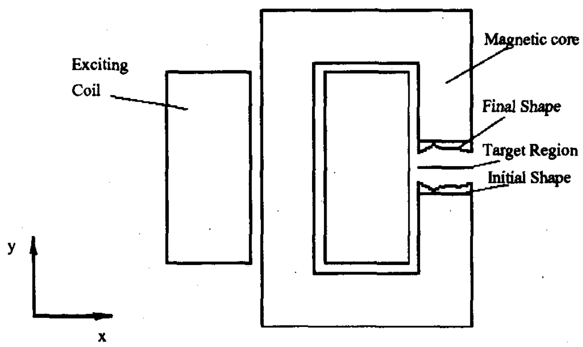


(c)

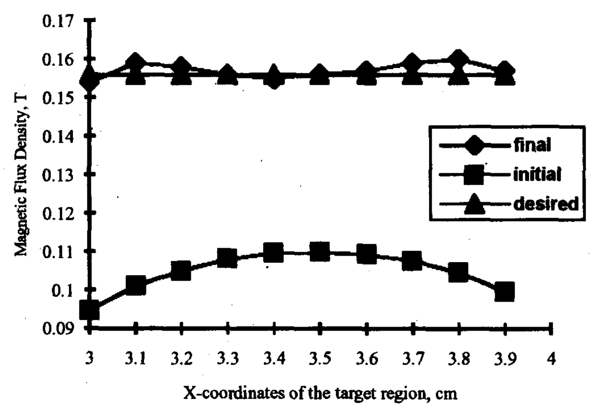
Fig. 1. Shape and location identification of magnetic material with different cross section shape. (a) Magnetic material with rectangular cross section shape. (b) Magnetic material with polygonal cross shape. (c) Magnetic material with triangular section shape.

The search for the magnetic material is carried out in the target region Ω with boundary L. Applying an external impressed field from an exciting coil the resultant magnetic flux density around the target region is measured. Here, we use inversions of numerically simulated experimental data, rather then real ones. For simulation we used BIEM with a true magnetic core geometry, and gave noise to the computed far field, thus producing input data to the inverse problem solver. The magnetic material is assumed to be with rectangular, polygonal and triangular cross-section shape. Figure 1 shows the true magnetic material shape, also the initial , intermediate and final shape of the magnetic material pattern in the searching process. As this figure shows, the final solution pattern of the magnetic material shape is sufficiently close to the true ones. It was found that the correct choice of the number of elements m to discretize the boundary L as well as the number of elements m_k with the best position of simple layer distribution of imaginary currents and which determine the new boundary L is very important in order to decrease number of steps in the searching process and to obtain the solution pattern. Also, the small changes of the magnetic material shape do not essentially affect to the far measurement field, especially when the sizes of magnetic material are much less than these of target region.

The results obtained designing the pole of the magnetic core in order to obtain the desired magnetic flux densities in the target region are shown in Fig. 2. The electromagnetic system under consideration has a closed magnetic circuit with an air gap. The pole shape is designed to obtain a given homogeneous magnetic flux density in the air gap. The magnetic flux density is tested at 10 points located in the middle of the air gap. The computations are carried out using BIEM. The initial shape of the pole as well as the final solution pattern are shown in Fig. 2(a). The distribution of magnetic flux density at initial and at final shapes as well as desired magnetic flux density are shown in Fig. 2(b). As is obvious from the figure, the distribution of the magnetic flux density in the target region converges to the desired magnetic flux density when the final pole shapes of the electromagnetic system are used.



(a)



(b)

Fig. 2. Pole design of magnetic core. (a) The initial shape of the pole and final solution pattern. (b) Distribution of magnetic flux density at initial and final shape of poles.

IV. Conclusion

The approach proposed above is easily applicable for shape and position identification of magnetic material in inaccessible regions from magnetic field it produced and for shape design of magnetic core in order to realise given magnetic field distribution. Based on the replacement of the magnetic core with simple layer distribution of imaginary currents this approach reduces the inverse problem of shape and location design to the inverse source problem. It was found that the pattern figure γ is quite useful in searching for the best position of the simple layer distribution of imaginary currents. The results obtained reveal that the approach is quite effective in the solution of magnetic core shape and location design problems.

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