Simulation and Design of Applied Electromagnetic Systems
T. Honma (Editor)

Massively parallel neural networks for the inverse source problems

K.Kitsuta and Y.Saito

Coll. of Engng., Hosei University, Kajino, Koganei, Tokyo 184, Japan

Abstract

We have previously proposed an approach to searching for the current distribution patterns in human heart. In the present paper, a theoretical background of our method is given by means of the neural networks. Our method yields the field source patterns having somewhat ambiguity so that it is required to pick up only the major patterns. This means that considerable experience is required to decide the major parts. In order to remove this difficulty, we propose here a new method that the magnetic and electric field source searchings are carried out in parallel by the massively parallel neural networks. After obtaining their independent results, a correlative analysis between them provides a highly accurate electromagnetic field source pattern.

1. INTRODUCTION

Most of the modern electronic medical systems are based on the applications of the inverse problems, for examples, X-ray computed tomography and ultrasonic imaging. Successful results of these first stage applications of the inverse problems are now leading to the second stage applications of the inverse problems, i.e. searching for the electric current paths in human heart and clarifying the neural behaviors in human brain. Definitely difference between the first and second stages is that the first stage is mainly concerned with the medium identification problems and the second is the electrical signal source searching problems , i.e. inverse source problems. Mathematically it has been proofed that the three dimensional current distributions in human body is not analytically evaluated by measuring the magnetic fields around the body. To overcome this difficulty, we have previously proposed an approach to searching for the current distribution patterns in human heart [1-3].

In the present paper, we show that a theoretical background of our method called "*Sampled Pattern Matching*" method can be interpreted as one of the neural networks. One of the weak points of our sampled pattern matching method is that the method yields the field source patterns having somewhat ambiguity. Thereby, it is required to pick up only the major patterns. This means that considerable experience is essentially required to decide the major parts. In order to remove this difficulty, we propose here a new method that the magnetic and electric field source searchings are carried out in parallel by the massively neural networks. After obtaining their independent results, a correlative analysis between them provides a highly accurate electromagnetic field source pattern. Thus, we have succeeded in realizing a new system for human heart diagnosis.

2. THE INVERSE SOURCE PROBLEM

2.1 System equations

Most of the electromagnetic field problems are reduced to solving the

64

following form equation:

$$\lambda \nabla^2 \psi = -\sigma, \quad (1)$$

where λ, ψ and σ are the medium parameter, potential and source density, respectively. Using the Green's function $G=1/(4\pi|r|)$, the integral form of (1) is

$$\psi = \int [\sigma/(4\pi|r|\lambda)]dv. \quad (2)$$

In the current fields, the parameters λ, ψ and σ in (1) correspond to the conductivity κ[s/m], electrical potential ϕ[V] and time derivative of electrical charge density $-\partial\rho/\partial t[C/(m^3 s)]$, respectively. By means of (2), the electric potential ϕ is

$$\phi = -\int [(\partial\rho/\partial t)/(4\pi|r|\kappa)]dv. \quad (3)$$

Discretizing (3) into the small subdivisions, we have

$$\mathbf{U} = \sum_{i=1}^{m} \alpha_i \mathbf{d}_i, \quad (4)$$

where

$$\left.\begin{array}{l} \mathbf{U} = [\phi_1, \phi_2, \cdots, \phi_n]^T, \\ \alpha_i = -[(\partial\rho_i/\partial t)/\kappa_i]\Delta v_i = P_{Ui}, \ i=1\sim m, \\ \mathbf{d}_i = [1/(4\pi)][(\mathbf{n}_i \cdot \mathbf{a}_{1i}/r_{1i}), (\mathbf{n}_i \cdot \mathbf{a}_{2i}/r_{2i}), \cdots, (\mathbf{n}_i \cdot \mathbf{a}_{ni}/r_{ni})] \\ \text{n:number of measured points,} \\ \text{m:number of subdivisions: } v=\Delta v_i + \Delta v_2 + \cdots + \Delta v_m. \end{array}\right] \quad (5)$$

In (5), $\alpha_i (=P_{Vi}[Vm])$, $\mathbf{n}_i$, $\mathbf{a}_{ji}$ and r_{ji} $(i=1\sim m, j=1\sim n)$ are the voltage dipole, unit vector in the direction of voltage dipole, unit directional vector from the source point i to measured point j and distance from the source point i to measured point j, respectively. Further, the electrical field vector **E** on the conductor surface can be obtained by taking the gradient of scalar potential ϕ, i.e. $\mathbf{E}=-\nabla\phi$.

By means of the Biot-Savart's law, the system equation of magnetic fields is written in much the same way as (4). In this case, the vectors and parameters are

$$\left.\begin{array}{l} \mathbf{U} = [H_1, H_2, \cdots, H_n]^T, \\ \alpha_i = |\mathbf{J}_i|\Delta v_i = P_{Ii}, \ i=1\sim m, \\ \mathbf{d}_i = [1/(4\pi)][(\mathbf{n}_i \times \mathbf{a}_{1i}/r_{1i}^2), (\mathbf{n}_i \times \mathbf{a}_{2i}/r_{2i}^2), \cdots, (\mathbf{n}_i \times \mathbf{a}_{ni}/r_{ni}^2)]^T. \end{array}\right] \quad (6)$$

In (6), $|\mathbf{J}_i|\Delta v_i$ $(= P_{Ii}[Am], i=1\sim m)$ is the current dipole where $\mathbf{J}_i$ is a current density in a volume Δv_i; $H_1, H_2, \cdots, H_n$ are the measured magnetic fields; and the others are similarly defined as those of (5). Generally, the field or potential measured region is limited in a some surface and the source region is limited in the volume enclosed by the measuring surface so that a following condition is commonly held on both systems.

$$m >> n. \quad (7)$$

2.2 Neural nets solutions

Key idea of the neural networks is that the parameters called weight functions of the networks having the multiple input and single output terminals are determined by the various input pattern vectors and their corresponding supervisor outputs. After finishing this learning process, any input pattern can be recognized and identified in accordance with the previous learning.

The leaning process of our problem is carried out by

Learning

INPUT		SUPERVISOR
		1 2 m
$\mathbf{d}_1'$	:	$\Gamma_1 = [1,0,..,0]^T$,
$\mathbf{d}_2'$	:	$\Gamma_2 = [0,1,..,0]^T$,
	....	
$\mathbf{d}_m'$	:	$\Gamma_m = [0,0,..,1]^T$,

where $\mathbf{d}_i' = \mathbf{d}_i / \| \mathbf{d}_i \|$ and Γ_i $(i=1,2,,\cdots,m)$ are the normalized input and supervisor vectors, respectively. The first synapse combination is analytically carried out by

$$W = \sum_{i=1}^{m} \Gamma_i (\mathbf{d}_i')^T = \begin{vmatrix} \mathbf{d}_1'^T \\ \mathbf{d}_2'^T \\ \cdot \\ \mathbf{d}_m'^T \end{vmatrix} \tag{8}$$

The input of the normalized vector $\mathbf{U}' = \mathbf{U}/\|\mathbf{U}\|$ to this perceptron W yields the first most dominant point h taking the maximum of output. Namely, threshold value of this perceptron is the maximum value in output $W\mathbf{U}'$. This corresponds to the first pilot point of the sampled pattern matching method [1,2]. Learning of the second perceptron unit is carried out in much the same way as those of the first perceptron unit:

Learning

INPUT		SUPERVISOR
		1 2 m-1
$(\mathbf{d}_h + \mathbf{d}_1)'$	:	$\Gamma_1' = [1,0,..,0]^T$,
$(\mathbf{d}_h + \mathbf{d}_2)'$	:	$\Gamma_2' = [0,1,..,0]^T$,
....		
$(\mathbf{d}_h + \mathbf{d}_m)'$	:	$\Gamma_m' = [0,0,..,1]^T$,

where h refers to the first perceptron output point;and $(\mathbf{d}_h+\mathbf{d}_j)' = (\mathbf{d}_h+\mathbf{d}_j)/\|\mathbf{d}_h+\mathbf{d}_j\|$ $(j=1,2,,\cdots,m-1;j\neq h)$ is a normalized input vector. The synapse combination is given by

$$W' = \sum_{j\neq h}^{m} \Gamma_j' (\mathbf{d}_h+\mathbf{d}_j)'^T = \begin{vmatrix} (\mathbf{d}_h + \mathbf{d}_1)'^T \\ (\mathbf{d}_h + \mathbf{d}_2)'^T \\ \cdots\cdots \\ (\mathbf{d}_h + \mathbf{d}_m)'^T \end{vmatrix}. \tag{9}$$

The input of $\mathbf{U}' = \mathbf{U}/\|\mathbf{U}\|$ to this second perceptron W' yields the second most dominant point g taking the maximum in the output $W'\mathbf{U}'$. This corresponds to the second pilot point of the sampled pattern matching method [1,2]. Thus, our sampled pattern matching method is one of the neural networks with following modifications: first, learning process with supervisor is analytically carried out. Second, threshold value is the maximum value not a constant. Third, same input vector for any unit.

Summation of the entire output $(W + W' + \cdots)\mathbf{U}'$ yields the normalized solution of (4) as

$$\left.\begin{aligned}
&\alpha_1[|\mathbf{d}_1|/|\mathbf{U}|] \simeq [\mathbf{U}/|\mathbf{U}|]^T\{[\mathbf{d}_1/|\mathbf{d}_1|]+[(\mathbf{d}_h+\mathbf{d}_1)/|\mathbf{d}_h+\mathbf{d}_1|]+\cdots\},\\
&\alpha_2[|\mathbf{d}_2|/|\mathbf{U}|] \simeq [\mathbf{U}/|\mathbf{U}|]^T\{[\mathbf{d}_2/|\mathbf{d}_2|]+[(\mathbf{d}_h+\mathbf{d}_2)/|\mathbf{d}_h+\mathbf{d}_2|]+\cdots\},\\
&\cdots\cdots\cdots\cdots\\
&\alpha_h[|\mathbf{d}_h|/|\mathbf{U}|] \simeq \{[\mathbf{U}/|\mathbf{U}|]^T[\mathbf{d}_h/|\mathbf{d}_h|]+1+1+\cdots\},\\
&\cdots\cdots\cdots\cdots\\
&\alpha_m[|\mathbf{d}_m|/|\mathbf{U}|] \simeq [\mathbf{U}/|\mathbf{U}|]^T\{[\mathbf{d}_m/|\mathbf{d}_m|]+[(\mathbf{d}_h+\mathbf{d}_m)/|\mathbf{d}_h+\mathbf{d}_m|]+\cdots\}.
\end{aligned}\right] \tag{10}$$

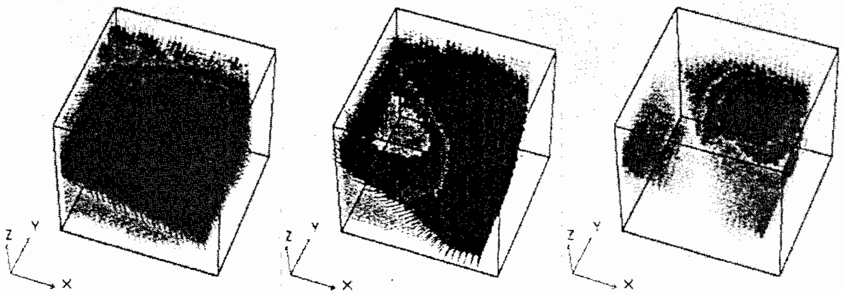
2.3 Massively parallel neural nets solutions

In physical system containing the field sources, it is possible to measure the different quantities caused by the same field sources. For example, in electromagnetic fields, we can measure the electric and magnetic fields distributions, independently. This leads to the massively parallel neural networks method. Namely, after evaluating the independent solutions for the electric and magnetic systems by their own neural networks, correlation between them may yield the electromagnetic field source distribution without a large amount of ambiguity.

Figures 1 and 2 show the voltage, current dipoles and correlation factor distributions in the human hearts [4]. Obviously, the correlation factor distributions in these figures demonstrate a definitely difference between the normal and abnormal heart conditions.

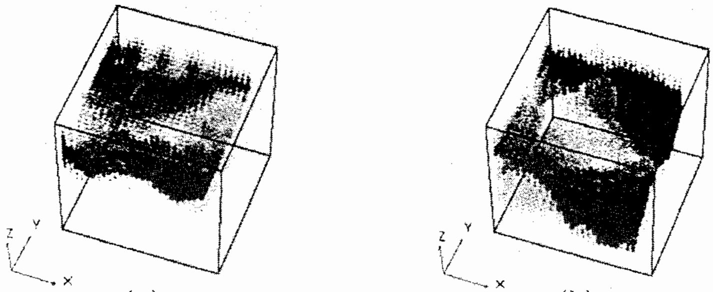
3. CONCLUSION

As shown above, we have clarified that our sampled pattern matching method is one of the neural networks. In order to evaluate the electromagnetic field source without ambiguity, we have proposed the massively parallel neural networks method. As a result, it has been suggested that the human heart diagnosis may be carried out with higher accuracy by combining the magnetocardiogram and electrocardiogram.



(a) (b) (c)

Fig.1 Normal human heart. (a)Estimated by the voltage dipole method using 6×6=36 points electric potentials measured at the top surface; (b) estimated by the current dipole method using 6×6=36 normal magnetic fields measured over 5mm above the top surface; and (c) correlation factor distribution obtained by taking the inner product between the current and voltage dipole vectors.



(a) (b)

Fig.2 Correlation factor distributions in the abnormal human hearts.
(a) Pulmonary hypertension and (b) Pulmonary stenosis.

4. REFERENCES

1 H.Saotome,et al, Trans. IEEJ, Vol.113-C, No.1, January(1993).
2 H.Saotome,et al, Trans. IEEJ, Vol.112-A, No.4, April(1992).
3 K.Kitsuta,et al., Proceedings of the 2nd Japan-Hungary Seminar, Ed. T.Honma et al, Hokkaido Univ. Press, Sep.(1992).
4 Y.Nakaya et al., Journal of Electrocardiology, Vol.21, No. 2 (1988).