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A designing strategy based on the inverse analysis

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Abstract
This paper proposes a new designing strategy based on the inverse analysis approach. As a concrete example, we try to decide the size and arrangement of the exciting coils in order to realize a locally concentrated magnetic field distribution along by the center axis for the local induction heating and hyperthermia use. As a result, it is found that the sampled pattern matching method previously proposed by us is a quite useful methodology for obtaining the desired magnetic field distributions.

1. INTRODUCTION

Conventional designing strategy of the magnetic devices, for example MRI magnet, magnetic shielding and electric machines, is that the desired magnetic field distribution is iteratively evaluated by solving a governing equation with electric current given condition.

On the contrary, in the present paper, the electric current distribution is estimated by the sampled pattern matching (SPM) method with the magnetic field distribution given condition [1-5]. This means that one of the designing strategies is proposed here by means of the inverse analysis approach. Conventional approach to the regular or forward problems yields a unique solution so that an iterative approach is essentially required to reaching a final goal of the designing. On the other side, our inverse approach based on the SPM method provides a unique solution pattern [2-5]. This means that our inverse approach allows some degree of the designer's creative freedom. As a concrete example, we try to decide the layout of the exciting coils in order to realize a locally concentrated magnetic field distribution along by the center axis for the hyperthermia and local induction heating use. As a result, it is found that our SPM method is a quite useful methodology for obtaining the desired magnetic field distributions.

2. THE INVERSE ANALYTICAL APPROACH FOR THE MAGNETIC DEVICES DESIGNING

2.1 Basic equations

Since most of the magnetic devices are composed of the loop windings, then let us consider a current filament in the form of a circle of radius R carrying the current I on the r-z cylindrical coordinate system. We are to find the magnetic field H at a general position of r and z. In the given case, the radially directed magnetic field H_r and the z-axis directed magnetic field H_z are respectively given by

$$H_r = \frac{I}{2\pi r}\frac{z}{\sqrt{(R+r)^2+z^2}}\left[\frac{R^2+r^2+z^2}{(R-r)^2+z^2}E(\kappa)-K(\kappa)\right] = If(r,z), \quad (1)$$

$$H_z = \frac{I}{2\pi}\frac{1}{\sqrt{(R+r)^2+z^2}}\left[\frac{R^2-r^2-z^2}{(R-r)^2+z^2}E(\kappa)+K(\kappa)\right] = Ig(r,z), \quad (2)$$

where $K(\kappa)$ and $E(\kappa)$ are the complete elliptic integrals of the first and second kinds, respectively [6]. Also κ in these integrals is defined by

$$\kappa = \sqrt{4rR/[(r+R)^2+z^2]}. \tag{3}$$

Let us consider a problem that the magnetic fields $H_{r1}, H_{r2}, \cdot, H_{rn}, H_{z1}, H_{z2}, \cdot, H_{zn}$ caused by the m-th unknown loop currents $I_1, I_2, \cdot, I_m$ are given quantities and we have to evaluate the m-th unknown loop currents with the condition $n \neq m$. Then by means of (2), it is possible to write a following system equation:

$$U = \sum_{i=1}^{m} I_i d_i, \tag{4}$$

where

$$U = [H_{r1}, H_{r2}, \cdot, H_{rn}, H_{z1}, H_{z2}, \cdot, H_{zn}]^T, \tag{5a}$$

$$d_i = [f(r_1,z_1), f(r_1,z_2), \cdot, f(r_n,z_n), g(r_1,z_1), g(r_1,z_2), \cdot, g(r_n,z_n)]^T, \tag{5b}$$

$i=1,2, \cdot, m.$

2.2 Sampled pattern matching method

<a>Algorism The formal algorism of our SPM method is as follows.

At first, we calculate a pattern matching figure γ_i using the column vector d_i of (5b) and given vector U of (5a), and then find the maximum pattern matching figure, i.e. if a point h takes the maximum of

$$\gamma_i = U^T \cdot d_i / [|U||d_i|], \quad i=1,2,\cdot,m, \tag{6a}$$

then we call h as a first pilot point and associated vector d_h is called the first pilot pattern vector.

The second step is carried out by combining the first pattern vector d_h and remaining pattern vectors in (5b), i.e. we search for the maximum of

$$\gamma_{hj} = U^T \cdot (d_h + d_j) / [|U||d_h + d_j|], \quad j=1,2,\cdot,m; \; j \neq h. \tag{6b}$$

If a point g takes the maximum in (6b) then g is the second pilot point and d_g is the second pilot pattern vector. Similar processes are continued up to the peak pattern matching figure γ.

<b>Solutions Generally, the number of equations n is not equivalent to the number of unknowns m so that it is difficult to obtain a unique solutions I_i $(i=1,2,\cdot,m)$ in (4). Obviously, Our SPM algorism of (6a) and (6b) has been based on the following assumptions.

At first, the solution I_i $(i=1,2,\cdot,m)$ in (4) takes the value of 1 or 0. This means that the pilot points take the unit value 1 and the other remaining points take the value 0. Secondly, each of the magnitudes I_i $(i=1,2,\cdot,m)$ in (4) could not be evaluated uniquely but is represented by the concentrating ratio of the unit value 1 in space. In the other words, a region comprising of a large number of unit currents represents a large current flowing region and a region comprising of a small number of unit currents represents a small current flowing region.

<c>Theoretical background Mathematically, the pattern matching figure γ_i in (6a) corresponds to the Cauchy-Schwarz relation so that the angle θ_i between the vectors U and d_i in a linear space is given by

$$\begin{aligned} \theta_i &= \cos^{-1}[\gamma_i] \\ &= \cos^{-1}\{U^T \cdot d_i / [|U||d_i|]\}, \quad i=1,2, \cdot, m. \end{aligned} \tag{7}$$

Obviously, (7) shows that the maximum value of γ_i corresponds to the minimum angle θ_i. Thereby, the maximum value of γ_i indicates which vector d_i $(i=1,2, \cdot, m)$, i.e. input source position, coincides with the vector U regardless of the input source magnitude.

In statistically, the operation of pattern matching corresponds to the factor analysis. Conventional factor analysis is always carried out only to the first step of our sampled pattern matching method. Second and third steps of our sampled pattern matching process are obviously generalization of the conventional factor analysis. Therefore, our sampled pattern matching method may be regarded as one of the generalized factor analysis methods.

The other theoretical background is the incomplete Fourier series. Equation (4) means that the vector U is a linear combination of the vectors $d_i (i=1 \sim m)$. Thereby, (4) can be modified into

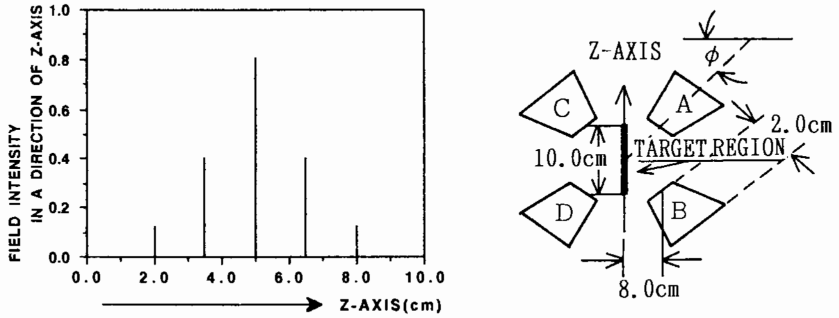
$$U = \sum_{i=1}^{m} \{ \beta_i d_i + \sum_{j \neq i}^{m} \{ \beta_i (d_i + d_j) + \sum_{k \neq i, k \neq j}^{m} \{ \beta_i (d_i + d_j + d_k) + \cdots \}\}\}. \qquad (8)$$

Each of the terms in (8) has its distinct field pattern. Namely, the 1st, 2nd and 3rd groups on the right give the one, two and three pairs of the N(north)-S(south) magnetic pole pairs, respectively. This means that the SPM method is one of the Fourier analysis methods if the vectors on the right of (8) are the orthogonal functions. However, they are not necessarily always the orthogonal function so that the SPM method is a kind of incomplete Fourier series [3].

2.3 An example

In order to demonstrate a concrete example of our designing strategy, let us consider a realization of a device giving a locally high magnetic field distribution along the center axis. Our problem is to realize the magnetic field distribution shown in Fig. 1(a) by the symmetrically arranged four winding. Figure 1(b) shows a schematic diagram of the device. The conditions of designing this device are as follows:

1) Target region is the 10cm line along with the z-axis in Fig. 1(b).
2) Each of the windings consists of the six loop coils.
3) The radius of each loop coil can be changed from 0.7 to 1.0cm.
4) The current of each loop coil has to take the same value.
5) The angle ϕ to the z-axis shown in Fig. 1(b) has to take the value within 30~80 degree.



(a) (b)

Figure 1. A device giving locally high magnetic field distribution. (a) Desired magnetic field distribution in the direction of z-axis. (b) Schematic diagram of the device.

The first step is to decide the most reasonable winding angle ϕ and the radius of each loop coil. The algorism of this step is as follows:

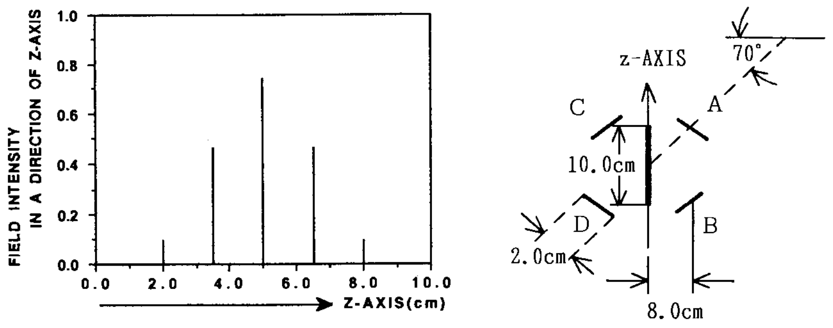
1) Set a winding angle ϕ_i within 30~80 degree.
2) Select one of the six loop coils of each winding.
3) Set a radius r_i of loop coil within 0.7~1.0cm.
4) Calculate the pattern matching figure γ_i by (6a) using the given field

vector U in Fig. 1(a) and pattern vector d_i in (5b).

5) Decide each of the six coil radii by selecting the maximum value of pattern matching figure.
6) Decide the most reasonable angle ϕ by selecting a loop coil taking the maximum value of pattern matching figure.

The second step is to decide which coil should be connected to the current source. This process was also carried out by the SPM procedure of (6a) and (6b) using the given field vector U in Fig. 1(a) as well as pattern vector d_i in (5b).

As a result, the magnetic field distribution shown in Fig. 2(a) was obtained. Figure 2(b) shows the schematic diagram of finally designed device. To summarizing our designing strategy, at the first step, the electric current distribution was estimated by the SPM method with the given magnetic field distribution. This lead to the position and shape determination of winding. At the second step, the coil connection was also determined by the SPM method in order to fit in excellently the given magnetic field distribution. Thus, our designing strategy proposed here needs not an iterative process. Obviously, a definitive difference between the conventional and our designing strategies is caused by the forward or inverse analytical approach.



(a) (b)

Figure 2. Results of the designing.
(a) Obtained field distribution corresponding to Fig. 1(a).
(b) Schematic diagram of the designed device.

3. CONCLUSION

As shown above, we have clarified that a completely different designing strategy to the conventional one is possible by means of the inverse analytical approach. One of the best merits of this new approach is that a fairly good result can be expected even if no iteration is carried out.

Thus, it has been suggested that the inverse analytical approach makes it possible to design the electromagnetic devices in a highly efficient manner.

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