Nonlinear Phenomena in Electromagnetic Fields
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ELECTROMAGNETIC FIELD SOURCE SEARCHING FROM THE LOCAL FIELD MEASUREMENTS

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Abstract

At first, we make a brief review of the inverse problems in the various fields. Secondly, we propose one of the methods for obtaining a unique solution pattern not a unique solution of the inverse problem in magnetostatic fields. Finally, we apply our method to the human heart diagnosis. As a result, the positions of defect exhibiting the Wolff-Parkinson-White syndrome are successfully identified by our approach.

1. INTRODUCTION

In 1823, Abel considered a shape identification problem of a frictionless hill. The problem: "Is it possible to identify the shape of hill by measuring the initial speed of a mass and round trip time to reach at the initial position ?" This problem could be reduced to solve the Abel's 1st kind integral equation, and successfully solved [1]. Another important problem under investigation from more than 100 years ago is the inverse source problems in gravimetry. The problem: "Is it possible to determine the earth's density by measuring the gravity on the earth surface ?" According to the works by Stokes (1867), Neumann (1906) and the others, it has been pointed out that the lack of information inside the earth results in ambiguity with regard to this inverse source problem [1]. Therefore, it is difficult to obtain a unique solution of the inverse source problems by the measurement of surface fields. But it should be noted that this problem is equivalent to the current density determination problem by measuring the local magnetic fields. In 1917, Radon gave a mathematical background of the computed tomography. After Oldendorf's experiment (1961) and Kuhl's experiment (1963), Hounsfield and Anbrose succeeded in realizing the computed tomography [1]. However, the computed tomography is not the inverse problem but one of the regular problems, because available data are not local but a set of orthogonal data around the target. The essential inverse problems in medicine are the electromagnetic field source identification from the electrocardiogram, magnetocardiogram, elctroencephalogram, magnetoencephalogram and the other electromagnetic data.

In the present paper, we propose one of the methods for obtaining a unique solution pattern not a unique solution of the inverse problem in magnetostatic fields. Finally, we apply our method to the human heart diagnosis. As a result, the positions of defect exhibiting the Wolff-Parkinson-White syndrome are successfully identified by our approach.

2. THE INVERSE SOURCE PROBLEM IN MAGNETOSTATIC FIELDS

2.1 System equation

The magnetic field **H** is related with the current density **J** by

$$H = \nabla\times\int [J/(4\pi|r|)]dv, \tag{1}$$

where r is a distance between the field **H** and source J points. In (1), the volume V containing the current density J is subdivided into a large number of subdivisions V_i, $i=1\sim m$, also the number of field points is denoted by n, then (1) reduces into

$$\mathbf{U} = \sum_{i=1}^{m} \alpha_i \mathbf{d}_i, \tag{2}$$

where

$$\mathbf{U} = [\mathbf{H}_1, \mathbf{H}_2, \cdots, \mathbf{H}_n]^T, \tag{3}$$

$$\mathbf{d}_i = [1/(4\pi)][\mathbf{n}_i \times \mathbf{a}_{1i}/r_{1i}^2, \mathbf{n}_i \times \mathbf{a}_{2i}/r_{2i}^2, \cdots, \mathbf{n}_i \times \mathbf{a}_{ni}/r_{ni}^2]^T, \tag{4}$$

$$\alpha_i = J_i V_i, \; i=1\sim m, \; m>>n. \tag{5}$$

In (4), $\mathbf{n}_i$ is a unit vector in the direction of J_i; $\mathbf{a}_{1i}, \mathbf{a}_{2i}, \cdots, \mathbf{a}_{ni}$ are the unit vectors from the source point i to the field points $1,2,\cdots,n$; $r_{1i}, r_{2i}, \cdots, r_{ni}$ are the distances from the source point i to the field points $1,2,\cdots,n$, respectively. Furthermore in (5), α_i, $i=1\sim m$, is a magnitude of the current dipole [2,3], also the condition $m>>n$ is always satisfied because the field **H** can be measured within the finite number of points. Equation (2) is a system equation of the inverse source problem.

2.2 Unique solution pattern searching

Equation (2) means that the vector **U** is a linear combination of the vectors $\mathbf{d}_i$, $i=1\sim m$. Thereby, (2) can be modified into

$$\mathbf{U} = \sum_{i=1}^{m}\{\beta_i \mathbf{d}_i + \sum_{j\neq i}^{m}\{\beta_{ij}(\mathbf{d}_i+\mathbf{d}_j) + \sum_{k\neq i, k\neq j}^{m}\{\beta_{ijk}(\mathbf{d}_i+\mathbf{d}_j+\mathbf{d}_k) + \cdots\}\}\}. \tag{6}$$

The 1st solution group in (6) is

$$\frac{\mathbf{U}^T\mathbf{d}_1}{|\mathbf{U}||\mathbf{d}_1|}, \frac{\mathbf{U}^T\mathbf{d}_2}{|\mathbf{U}||\mathbf{d}_2|}, \cdot\cdot, \frac{\mathbf{U}^T\mathbf{d}_h}{|\mathbf{U}||\mathbf{d}_h|}, \cdot\cdot, \frac{\mathbf{U}^T\mathbf{d}_m}{|\mathbf{U}||\mathbf{d}_m|}. \tag{7a}$$

If the term $\mathbf{U}^T\mathbf{d}_h/[|\mathbf{U}||\mathbf{d}_h|]$ in (7a) takes the maximum, then 2nd solution group is

$$\frac{\mathbf{U}^T(\mathbf{d}_h+\mathbf{d}_1)}{|\mathbf{U}||\mathbf{d}_h+\mathbf{d}_1|}, \frac{\mathbf{U}^T(\mathbf{d}_h+\mathbf{d}_2)}{|\mathbf{U}||\mathbf{d}_h+\mathbf{d}_2|}, 1, \cdot\cdot, \frac{\mathbf{U}^T(\mathbf{d}_h+\mathbf{d}_m)}{|\mathbf{U}||\mathbf{d}_h+\mathbf{d}_m|}. \tag{7b}$$

Similar process is continued until the peak value of inner product can be obtained. Thereby, the normalized solutions of (2) are

$$\alpha_1|\mathbf{d}_1| \simeq \mathbf{U}^T\{[\mathbf{d}_1/|\mathbf{d}_1|] + [(\mathbf{d}_h+\mathbf{d}_1)/|\mathbf{d}_h+\mathbf{d}_1|] + \cdots\}, \tag{8a}$$
$$\alpha_2|\mathbf{d}_2| \simeq \mathbf{U}^T\{[\mathbf{d}_2/|\mathbf{d}_2|] + [(\mathbf{d}_h+\mathbf{d}_2)/|\mathbf{d}_h+\mathbf{d}_2|] + \cdots\}, \tag{8b}$$

....

$$\alpha_h|\mathbf{d}_h| \simeq \mathbf{U}^T\{[\mathbf{d}_h/|\mathbf{d}_h|] + 1 + 1 + \cdots\}, \tag{8c}$$

....

$$\alpha_m|\mathbf{d}_m| \simeq \mathbf{U}^T\{[\mathbf{d}_m/|\mathbf{d}_m|] + [(\mathbf{d}_h+\mathbf{d}_m)/|\mathbf{d}_h+\mathbf{d}_m|] + \cdots\}. \tag{8d}$$

Obviously, this method has assumed that the angle between the vectors $\mathbf{d}_i$ and $\mathbf{d}_j$ $(i\neq j)$ is always smaller than $\pi/2$, so that this gives a unique solution pattern not the exact solutions [4,5].

2.3 Examples

Figures 1(a) and 1(b) show an exact current distribution in a cubic and its accompanying magnetic field distribution pattern normal to the top surface of the cubic, respectively. Figure 1(c) shows a most dominant pattern (top 10%) in the current distribution computed from the field of Fig. 1(b).

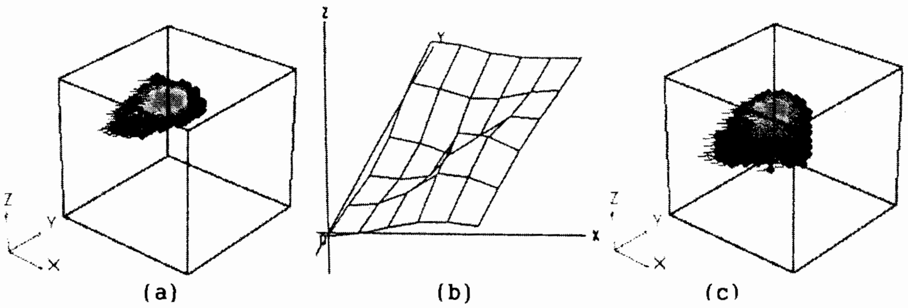


Fig.1 (a)Exact current distribution in a cubic and (b) its accompanying magnetic field distribution pattern normal to the top surface of the cubic. (c) Computed current distribution (top 10%), where m=395352 and maximum error between the measured and computed fields is 0.5%.

Figures 2(a) and 2(b) show an exact current distribution in a cubic and its accompanying magnetic field distribution pattern normal to the top surface of the cubic, respectively. Figure 2(c) shows a most dominant pattern (top 10%) in the current distribution computed from the field of Fig. 2(b).

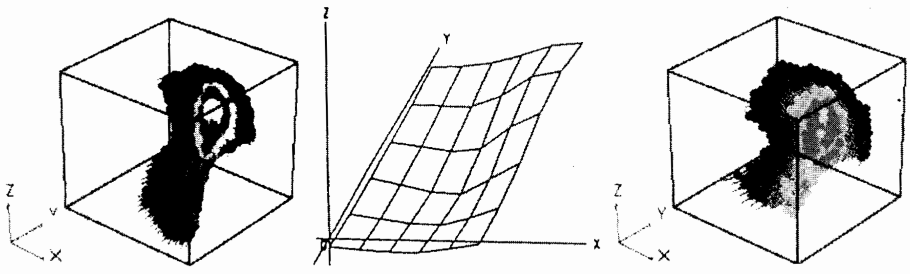
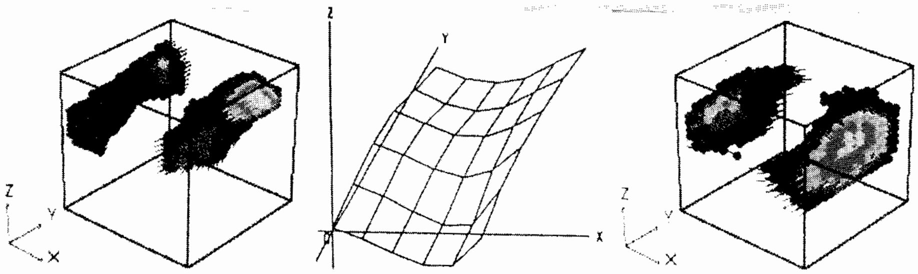


Fig.2 (a)Exact current distribution in a cubic and (b) its accompanying magnetic field distribution pattern normal to the top surface of the cubic. (c) Computed current distribution (top 10%), where m=395352 and maximum error between the measured and computed fields is 0.01%.

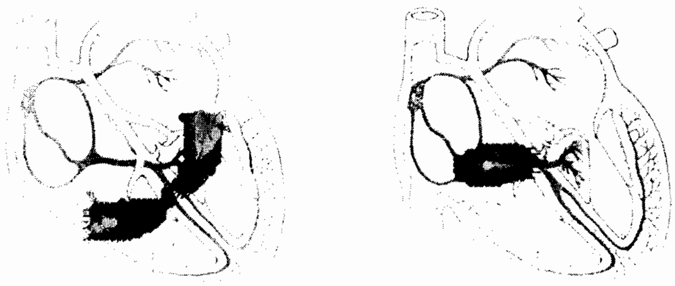
Figures 3(a) and 3(b) show an exact current distribution in a cubic and its accompanying magnetic field distribution pattern normal to the top surface of the cubic, respectively. Figure 3(c) shows a most dominant pattern (top 10%) in the current distribution computed from the field of Fig. 3(b). The CPU time for any examples using the 25 MIPS computer was about 15 minutes. Further, it is obvious that as possible as large number of freedom m and measured points n are reasonable to get a higher accurate result. Thus, it has been verified that our method provides the unique solution vector patterns corresponding to the exact ones.



(a) (b) (c)

Fig.3 (a)Exact current distribution in a cubic and (b) its accompanying magnetic field distribution pattern normal to the top surface of the cubic. (c) Computed current distribution (top 10%), where m=395352 and maximum error between the measured and computed fields is 0.36%.

Finally, we applied our method to the diagnosis of the human heart exhibiting WPW syndrome. The computed results in figures 4(a) and 4(b) suggest that the defect positions are the Kent and James bundles.



(a) (b)

Fig.4 (a) Kent, and (b) James bundle defects in human heart.

3. CONCLUSION

As shown above, it is difficult to obtain the exact solutions of the inverse source problems in magnetostatic fields. But it is possible to obtain the unique solution vector patterns by our method. As a result, the positions of defect in human heart exhibiting the WPW syndrome have been identified by the local magnetic field measurements.

4.REFERENCES

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