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A REPRESENTATION OF MAGNETIZATION CHARACTERISTICS FOR COMPUTATIONAL MAGNETODYNAMICS*

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In order to evaluate the magnetodynamic fields exactly, we derive a Chua type magnetization model based on the magnetic domain theory. Also, we examine the frequency characteristics of this Chua type model. As a result, it is revealed that the Chua type model has a versatile ability to represent the typical low and high frequency magnetization characteristics.

1. Introduction

With the development of modern high speed super computers as well as the high performance personal computers, numerical methods, e.g. finite element, finite differences and integral approaches, become one of the deterministic methods for analysing and designing electromagnetic devices. In order to calculate the magnetic fields exactly, it is essentially required to work out a macroscopic constitutive equation representing the magnetization characteristics of the ferromagnetic materials. Because of their physical structure, the ferromagnetic materials exhibit various magnetization features such as saturation, hysteresis, anisotropy, aftereffect, magnetostriction, frequency dependence, mechanical stress dependence and temperature dependence [1]. Among these properties, at least, it may be taken into account the saturation, hysteresis, aftereffect and frequency dependence for the computational analysis and design of the electromagnetic devices. Even though a more general theoretical background of magnetization has been reported by Maugin [2], because of simplicity, we derive here a Chua type magnetization model based on a simple bar like domain walls model. By means of this model, a versatile capability of the Chua type model is demonstrated.

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2. Mechanism of magnetization process

2.1. Magnetic domains

Many of the phenomena of the magnetization curve and hysteresis loop can be described to the advantage in terms of the magnetic domain theory. The theory that a ferromagnetic material is composed of many regions, and each of the regions is magnetized to saturation in some directions, was first stated by Weiss in 1907 [1]. The existence of such domains can be observed by the study of powder patterns and other means such as optical effects [3]. In demagnetized state, the directions in which the domains are saturated are either distributed at random or in some ways such that the resultant magnetization of the specimen as a whole is zero. Application of an external field changes only the direction of magnetization in a given volume, not the magnitude.

A typical initial magnetization curve, as shown in fig. 1, may be divided into three major parts. In the first part, the curve starts from origin and rises upward holding the following

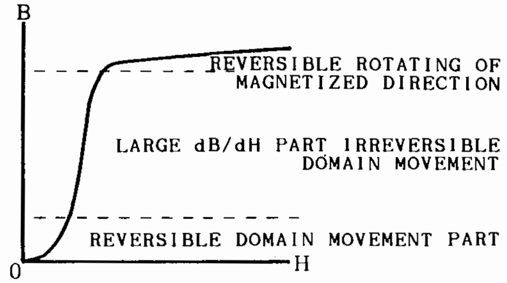


Fig. 1. A typical initial magnetization curve and its three major parts.

relation:

$$B = \mu_i H + \tfrac{1}{2}\nu H^2 , \quad (1)$$

where B, H, μ_i and ν are the flux density, field, initial permeability and Rayleigh's constant [3]. According to the observation at this part, it has been clarified that the magnetization is carried out by the reversible domain movement. On the contrary, the magnetization at the second part taking a large dB/dH is carried out by the irreversible domain movement. Further, the magnetization at the third part on the initial magnetization curve in fig. 1 is carried out by the reversible rotation of magnetized direction.

Thus, most of the magnetization characteristics can be explained in terms of the magnetic domain theory.

2.2. A Chua type magnetization model

To derive a constitutive equation representing the typical magnetization characteristics, let us consider a simple bar like domain walls model shown in fig. 2. When an external field H_s is applied to this domain, the following relationship can be established

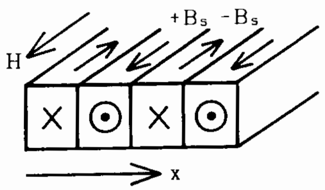


Fig. 2. Bar like magnetic domain walls model.

$$B = \mu_0 H_s + nB_s , \quad (2a)$$

$$= \mu_0[1 + (nB_s/\mu_0 H_s)]H_s , \quad (2b)$$

$$= \mu H_s , \quad (2c)$$

where B_s, n, μ_0 and μ are the saturation flux density in each of the domains, number of domains in accordance with the direction of externally applied field H_s, permeability of air and permeability of the specimen, respectively.

Since the magnetization model exhibits various magnetization characteristics as the solutions of model with the reasonable initial and boundary conditions, therefore, the model itself must be composed of the parameters not affected by the past histories. One of the unique properties independent of the past histories is an ideal or anhysteretic magnetization curve. The ideal magnetization curve is measured by first applying the superposed static and alternating fields, and then reducing the alternating field to zero and observing the flux density.

Differentiation of eq. (2a) with time t yields the following relation:

$$\frac{dB}{dt} = \mu_0 \frac{dH}{dt} + B_s \frac{dn}{dt} , \quad (3a)$$

$$= \left[\mu_0 + B_s \frac{\partial n}{\partial H}\right] \frac{dH}{dt} + B_s \frac{\partial n}{\partial x} \frac{dx}{dt} , \quad (3b)$$

$$= \mu_r \frac{dH}{dt} + B_s \frac{\partial n}{\partial x} v , \quad (3c)$$

where H, v and μ_r denote the applied field, velocity (dx/dt) of domain movement and reversible permeability, respectively. Consideration of eqs. (3a)–(3c) means that the magnetization of ferromagnetic material corresponds to the time variation of field intensity, dH/dt, and physical domain movement, dx/dt. That is, the induced voltage per unit area, dB/dt, is composed of the transformer and velocity induced voltages.

When a hysteresis coefficient s [Ohm/m] is introduced into the relations (3a)–(3c), the magnetic field H_d due to the domain movement is given by

$$H_d = \frac{1}{s} B_s \frac{\partial n}{\partial x} v , \tag{4a}$$

$$= \frac{1}{s}\left[\frac{dB}{dt} - \mu_r \frac{dH}{dt}\right]. \tag{4b}$$

Summation of the static field H_s in eq. (2c) and dynamic field H_d in eq. (4a) or (4b) gives a general field H as

$$H = H_s + H_d , \tag{5a}$$

$$= \frac{1}{\mu} B + \frac{1}{s} B_s \frac{\partial n}{\partial x} v , \tag{5b}$$

$$= \frac{1}{\mu} B + \frac{1}{s}\left[\frac{dB}{dt} - \mu_r \frac{dH}{dt}\right]. \tag{5c}$$

Equation (5b) or (5c) is a Chua type model [4–8]. The hysteresis coefficient s in eqs. (4a)–(5c) physically corresponds to a frictional coefficient between the domain walls so that the loss is caused by mechanical friction. This frictional loss is classified into two major components: one is a static frictional loss which is proportional to the velocity v of a domain movement, and the other is a dynamic frictional loss which is proportional to v^2. These static and dynamic losses are respectively known as the hysteresis and anomalous eddy current losses, because the velocity v of the domain movement is proportional to the exciting frequency f [6–8].

According to refs. [4, 5], a relationship between the Rayleigh's constant ν (which is equivalent to the Preisach's function in a low field) in eq. (1) and hysteresis coefficient s in the Chua type model (5b) or (c) is given by

$$s = \nu \frac{dH}{dt} , \tag{6}$$

where a weakly magnetized region has been assumed. Substituting eq. (6) into the modified form of eq. (5c) yields

$$H + \frac{\mu_r}{\nu} = \frac{1}{\mu} B + \frac{1}{\nu}\frac{\partial B}{\partial H} . \tag{7}$$

An initial magnetization curve in a weakly magnetized region can be obtained as a solution of (7):

$$B = \mu H + \frac{\mu}{\nu}(\mu_r - \mu)[1 - \exp(-H\nu/\mu)] , \tag{8a}$$

$$\simeq \mu_r H + \tfrac{1}{2}\nu H^2 , \tag{8b}$$

where $\mu_r \ll \mu$ and $\exp(-H\nu/\mu) \simeq 1 - (H\nu/\mu) + (1/2)(H\nu/\mu)^2$ are assumed. The reversible permeability μ_r in eq. (8b) is reduced to the initial permeability μ_i in eq. (1) on the initial magnetization curve so that it is obvious that the Chua type model exhibits the Rayleigh's curve.

The permeability μ in eq. (5c) is obtained from the ideal magnetization curve as mentioned before. Also, the reversible permeability μ_r is obtained accompanying the measurement of the ideal magnetization curve as described in ref. [7]. On the other hand, the hysteresis coefficient s in eq. (5c) is measured by setting the $B = 0$ condition. This means that the parameters μ and μ_r take a constant value so that the hysteresis coefficient s is obtained by the measurements of dB/dt and dH/dt in eq. (5c). An example of these parameters are shown in fig. 3 for a ferrite (TDK H5c2).

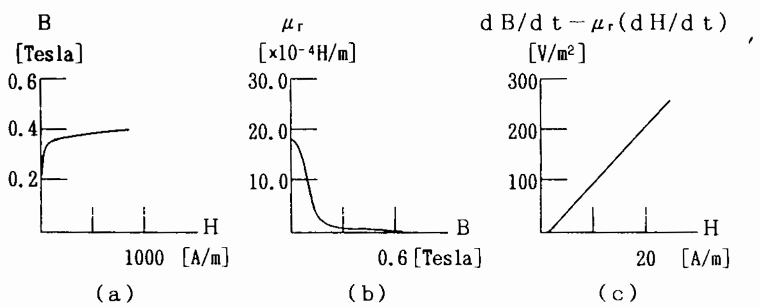


Fig. 3. Parameters μ, μ_r and s of the Chua type model for a ferrite (TDK H5c2); (a) permeability $\mu(=B/H)$, (b) reversible permeability μ_r vs. B curve, where B is a bias flux density, (c) $(dB/dt) - \mu_r(dH/dt)$ vs. H curve, where $s = [(dB/dt) - \mu_r(dH/dt)]/H$.

2.3. Frequency characteristics of the model

At low frequency, the flux density B takes a relatively high value so that the nonlinearities of parameters μ, μ_r and s must be fully taken into account in the computation. Examples of low frequency hysteresis loops are shown in fig. 4. Because of a strong influence of the characteristic time (like viscoelasticity), the overall difference between the computed and experimented loops is observed.

For cases in which, the flux density B is sufficiently small, the flux density B and field intensity H are approximately sinusoidally time varying. This makes it possible to employ a complex permeability representation of the magnetization characteristics. The complex permeability is simply derived by assuming the constants μ, μ_r and s in eq. (5c). When we employ a complex notation $\mathrm{d}/\mathrm{d}t = \mathrm{j}\omega$, the complex permeability $\mu(\omega)$ is defined by

$$\mu(\omega) = B/H \,, \tag{9a}$$

$$= \mu_R(\omega) - \mathrm{j}\mu_I(\omega) \,, \tag{9b}$$

$$= \mu[(s^2 + \omega^2\mu\mu_r)/(s^2 + \omega^2\mu^2)] - \mathrm{j}\mu\omega s[(\mu - \mu_r)/(s^2 + \omega^2\mu^2)] \,, \tag{9c}$$

where $\mathrm{j} = \sqrt{-1}$ and $\omega = 2\pi f$ (f = frequency).

The parameters μ and μ_r in eq. (9c) are determined by taking the frequency limits of $\mu_R(\omega)$ in eq. (9b). That is $\mu_R(0) = \mu$ and $\mu(\infty) = \mu_r$. Further, the hysteresis coefficient s in eq. (9c) is determined by $s = \omega_p\mu$, where ω_p is an angular frequency taking the peak value of $\mu_I(\omega)$ in eq. (9b).

Figure 5 shows an example of the frequency characteristics of complex permeability.

Finally, we applied our model to a simple two-dimensional magnetodynamic field problem. The governing equations are

$$\nabla \times E = -\mathrm{j}\omega B \,, \tag{10a}$$

$$\nabla \times H = J \,, \tag{10b}$$

$$J = \kappa E \,, \tag{10c}$$

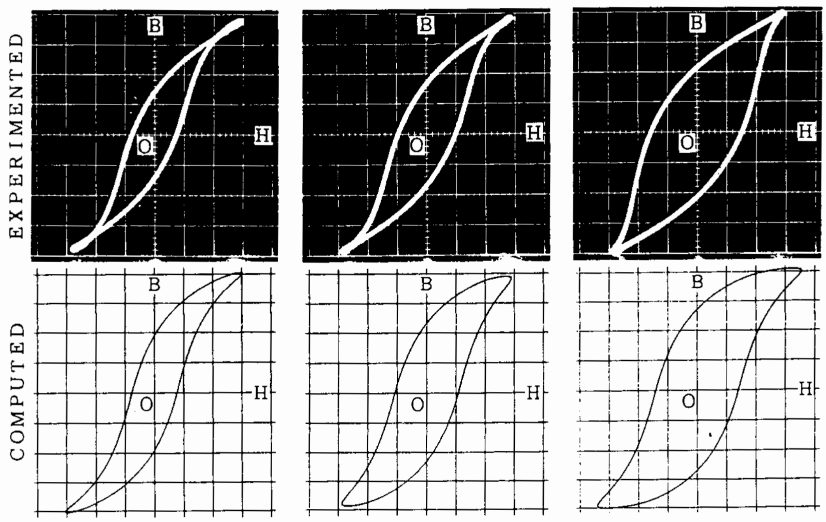
$$B = \mu(\omega)H \,, \tag{10d}$$

where E, J and κ are the electric field, current density and conductivity, respectively. Combining eq. (5c) with eqs. (10a)–(10d) yields the following partial differential equation:

$$\nabla^2 B + \alpha B = 0 \,, \tag{11a}$$

where

$$\alpha = \omega\kappa\mu \, \frac{\omega\mu_r - \mathrm{j}s}{s + \mathrm{j}\omega\mu} \,. \tag{11b}$$



(a) (b) (c)

B : 0. 0 6 5 [T/div.] H : 7. 7 1 [(A/m)/div.]

Fig. 4. Examples of low frequency hysteresis loops. Ferrite (TDK H5c2), (a) 50 Hz, (b) 100 Hz, (c) 200 Hz.

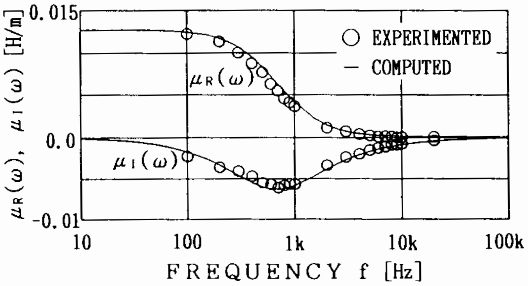


Fig. 5. Frequency characteristic of the complex permeability for the ferrite (H5c2).

Equation (11a) is a complex homogeneous Helmholtz equation and can be solved by the conventional finite element method [9]. The boundary condition at the interface between the exciting coil and core surface is given by

$$\left.\frac{B\varepsilon^{j\omega t}}{\mu(\omega)}\right|_{\text{core}} = H_m \varepsilon^{j\omega t}\Big|_{\text{coil}}, \tag{12}$$

As shown in fig. 6, the problem region is a cross section of the tested ferrite core having conductivity $\kappa = 6.665[\text{S/m}]$ and $1\,\text{cm} \times 1\,\text{cm}$ cross sectional area. Figures 7(a)–(e) show the experimental hysteresis loops at a frequency of 1 kHz, 3 kHz and 10 kHz, respectively. Also, figs. 7(b)–(f) show the corresponding finite element solutions of hysteresis loops at a frequency of 1 kHz, 3 kHz and 10 kHz, respectively. The horizontal positions in figs. 7(b)–(f) correspond to those on the diagonal line drawn from outer to inner in fig. 6. From these results, it is obvious that the magnetic field is uniformly distributed

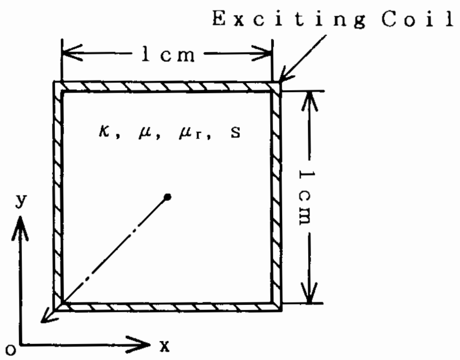


Fig. 6. Two dimensional problem region (a cross section of a toroidal core).

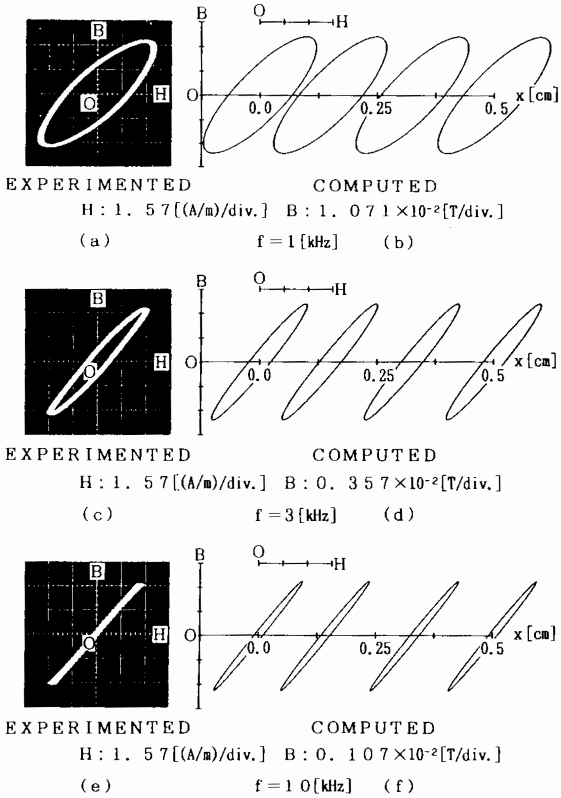


Fig. 7. Hysteresis loops for the ferrite (TDK H5c2); (left) experimental loops (average flux density vs. field), (right) computed local loops along the diagonal line in fig. 6.

over the problem region. This means that the frequency dependence of the loops is mainly dominated by the essential frequency characteristic of complex permeability not the field distribution effects such as a skin effect.

3. Conclusion

As shown above, we have derived the Chua type magnetization model and examined its frequency characteristics. As a result , it has been revealed that the Chua type model is capable of representing various frequency characteristics. This Chua type model is a fairly simple differential form so that it may be considered as one of the best representations for computational magnetodynamics.

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