A magnetization model for computational magnetodynamics

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In order to calculate the magnetodynamic fields exactly, it is essential to work out a magnetization model. We have previously proposed a Chua-type magnetization model based on magnetic domain theory. This Chua-type model is now applied to typical ferromagnetic materials, such as iron, ferrite, and amorphous magnetic material. As a result, it is revealed that the typical magnetization characteristics of representative ferromagnetic materials can be satisfactory reproduced by our Chua-type model.

I. INTRODUCTION

In order to calculate the magnetic fields, it is required to work out a macroscopic constitutive equation representing the magnetization characteristics of ferromagnetic materials. Because of their physical structure, the ferromagnetic materials exhibit various magnetization features including saturation, hysteresis, anisotropy, aftereffect, magnetostriction, frequency dependence, mechanical stress dependence, and temperature dependence.[1,2]

In the present paper, we derive a Chua-type magnetization model based on a simple barlike domain-wall model. Also, it is found that a similar model can be derived by considering a magnetic aftereffect. Further, it is shown that a mathematical model derived by Hodgedon is one of the Chua-type models.[3] This Chua-type model is now applied to typical ferromagnetic materials, such as iron, ferrite, and amorphous magnetic material. As a result, it is revealed that the typical magnetization characteristics of representative ferromagnetic materials can be satisfactory reproduced by our Chua-type model.

II. THE MAGNETIZATION MODEL

A. Domain-based model

To derive a constitutive equation representing typical magnetization characteristics, let us consider a simple barlike domain-wall model shown in Fig. 1. When an external field H_s is applied, then the following relationship can be established:

$$B=\mu_0 H_s + nB_s=\mu_0\left(1+\frac{nB_s}{\mu_0 H_s}\right)H_s=\mu H_s, \tag{1}$$

where B_s, n, μ_0, and μ are the saturation flux density in each of the domains, number of domains in accordance with the direction of externally applied field H_s, permeability of air, and permeability of the specimen, respectively. The magnetization model should exhibit various magnetization characteristics, such as a hysteretic property, as the solutions of the model. This means that the model itself must be composed of parameters not affected by past histories. One of the unique properties independent of the past history is an ideal or anhysteretic magnetization curve. If the relation (1) has been established for the ideal magnetization curve, then obviously these relations represent a static magnetization characteristic corresponding to each of the domain situations. This means that the permeability μ is obtained from the ideal magnetization curve. Differentiation (1) with time t yields a following relation:

$$\frac{dB}{dt}=\mu_0\frac{dH}{dt}+B_s\frac{dn}{dt} \tag{2a}$$

$$=\left(\mu_0+B_s\frac{\partial n}{\partial H}\right)\frac{dH}{dt}+B_s\frac{\partial n}{\partial x}\frac{dx}{dt} \tag{2b}$$

$$=\mu_r\frac{dH}{dt}+B_s\frac{\partial n}{\partial x}v, \tag{2c}$$

where H, v, and μ_r denote the applied field, velocity (dx/dt) of domain movement, and reversible permeability, respectively. Equations (2a)–(2c) are valid as long as the specific magnetization mode is maintained.

Consideration of (2a)–(2c) suggests that the induced voltage per unit area dB/dt is composed of the transformer and velocity-induced voltages. When a hysteresis coefficient s (Ω/m) is introduced into the relations (2a)–(2c), then the magnetic field H_d due to the domain movement is given by

$$H_d=\frac{1}{s}B_s\frac{\partial n}{\partial x}v=\frac{1}{s}\left(\frac{dB}{dt}-\mu_r\frac{dH}{dt}\right), \tag{3}$$

where it has been assumed that the width of the domains is fixed and only their number changes as the medium becomes magnetized. Summation of the static field H_s in (1) and dynamic field H_d in (3) gives a general field H as

$$H=H_s+H_d \tag{4a}$$

$$=\frac{1}{\mu}B+\frac{1}{s}B_s\frac{\partial n}{\partial x}v \tag{4b}$$

$$=\frac{1}{\mu}B+\frac{1}{s}\left(\frac{dB}{dt}-\mu_r\frac{dH}{dt}\right). \tag{4c}$$

Equation (4b) or (4c) is a domain-based Chua-type model.[4–8] The hysteresis coefficient s in (3) physically corresponds to a friction coefficient between the domain walls so that the loss is caused by mechanical friction.

The frictional loss is classified into two major components: One is a static frictional loss which is proportional to the velocity v of domain movement, and the other is dynamic frictional loss which is proportional to v^2. These static and dynamic losses are, respectively, known as the hysteresis and anomalous eddy current losses, because the velocity v of domain movement is proportional to the exciting frequency f.[6–8] The permeability μ in (4c) is ob-

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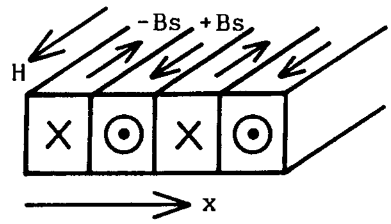

FIG. 1. Bar-like magnetic domain-wall model.

tained from the ideal magnetization curve as mentioned before. Also, the reversible permeability μ_r is obtained accompanying the measurement of the ideal magnetization curve as described in Ref. 4. On the other side, the hysteresis coefficient s in (4c) is measured by setting the $B=0$ condition. This means that the parameters μ and μ_r take constant values so that the hysteresis coefficient s is obtained by the measurements of dB/dt and dH/dt in (4c).

B. Aftereffect-based model

With τ denoting a relaxation time, a magnetic aftereffect can be represented by

$$M=\chi_m H(1-\epsilon^{-t/\tau}), \tag{5}$$

where M, H, and χ_m are the magnetization, magnetic field, and susceptibility, respectively.[2] When we differentiate M with time t, we then have

$$\frac{dM}{dt}=\frac{1}{\tau}\chi_m H\epsilon^{-t/\tau}=\frac{1}{\tau}(\chi_m H-M). \tag{6}$$

A general relationship among the flux density B, field H (not constant), and magnetization M is given by

$$B=\mu_0 H+\mu_0 M \quad \text{or} \quad M=(B/\mu_0)-H. \tag{7}$$

Substituting (7) into (6), we can obtain

$$H=\frac{1}{\mu}B+\frac{\tau}{\mu}\left(\frac{dB}{dt}-\mu_0\frac{dH}{dt}\right), \tag{8}$$

where $\mu=\mu_0(1+\chi_m)$. Equation (8) is a magnetic aftereffect-based model, which is similar in form to the domain-based Chua-type model (4c). A relationship between them is that the reversible permeability μ_r and hysteresis coefficient s in (4c) correspond to the permeability of air μ_0 and μ/τ in (8), respectively.

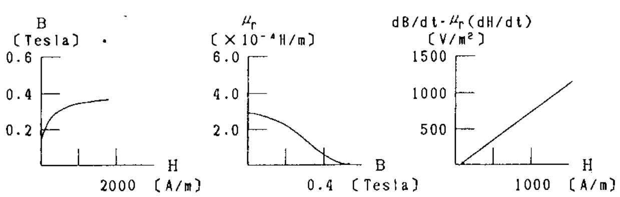

FIG. 2. An example of parameters. (a) Ideal magnetization curve $\mu=B/H$, (b) reversible permeability μ_r-vs-B curve, where B is a bias flux density, and (c) $(dB/dt)-\mu_r(dH/dt)$-vs-H curve, $s=[(dB/dt)-\mu_r(dH/dt)]/H$.

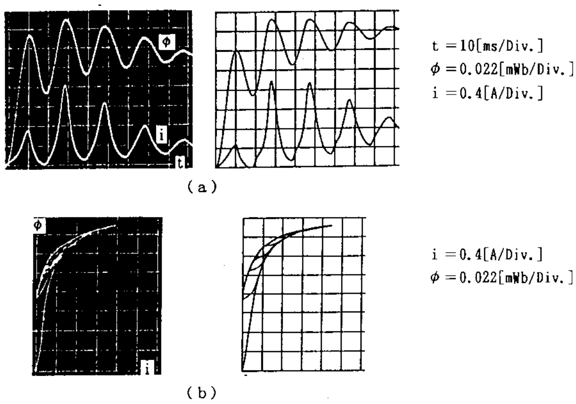

FIG. 3. Anhysteretic magnetization characteristics of a laminated iron core. (a) Time variations of the flux ϕ and current i; (b) hysteresis loops. Left: experimental and right: computed.

C. Mathematical-based model

In 1988, Hodgedon derived a mathematical model of magnetic hysteresis:

$$\frac{dH}{dt}=\alpha\left|\frac{dB}{dt}\right|\,|f(B)-H|+g\left(B,\frac{dB}{dt}\right)\frac{dB}{dt}, \tag{9}$$

where α is a parameter depending on the material, f is a single-valued function of B, and g is a single-valued function of B and dB/dt.[3] In (9), let us assume $f(B)>H$; we then have

$$H=f(B)+\left[g\left(B,\frac{dB}{dt}\right)\Big/\alpha\left|\frac{dB}{dt}\right|\right]\times\left\{\frac{dB}{dt}-\left[1/g\left(B,\frac{dB}{dt}\right)\right]\frac{dH}{dt}\right\}. \tag{10}$$

Comparison of (10) with the domain-based Chua-type model (4c) reveals the following relationships: $f(B)=(1/\mu)B$, $1/g(B, dB/dt)=\mu_r$, and $g(B, dB/dt)/\alpha|dB/dt|=1/s$. Thus, it is obvious that the mathematical-based model (10) is one of the domain-based Chua-type models (4c).

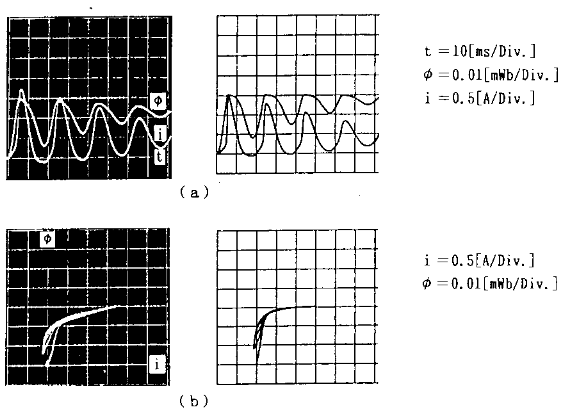

FIG. 4. Anhysteretic magnetization characteristics of a ferrite core (TDK K6A). (a) Time variations of the flux ϕ and current i; (b) hysteresis loops. Left: experimental and right: computed.

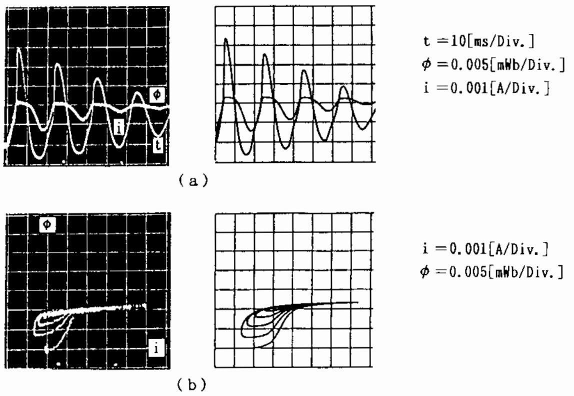

FIG. 5. Anhysteretic magnetization characteristics of an amorphous core (Toshiba MA1). (a) Time variations of the flux ϕ and current i; (b) hysteresis loops. Left: experimental and right: computed.

III. TYPICAL MAGNETIZATION CHARACTERISTICS

A. Rayleigh's curve

According to Refs. 4 and 5, a relationship between Rayleigh's constant ν (which is equivalent to the Preisach's function in a low field) and hysteresis coefficient s in the domain-based Chua-type model (4b) or (4c) is given by $s=\nu(dH/dt)$, where a weakly magnetized region has been assumed. Substituting this relation into the modified form of (4c) yields

$$H+\frac{\mu_r}{\nu}=\frac{1}{\mu}B+\frac{1}{\nu}\frac{\partial B}{\partial H}. \qquad (11)$$

An initial magnetization curve in a weakly magnetized region can be obtained as a solution of (11):

$$B=\mu H+\frac{\mu}{\nu}(\mu_r-\mu)\left[1-\exp\left(-\frac{H\nu}{\mu}\right)\right]$$
$$\simeq\mu_r H+\frac{1}{2}\nu H^2, \qquad (12)$$

where $\mu_r \ll \mu$ and $\exp(-H\nu/\mu)\simeq 1-(H\nu/\mu)+\frac{1}{2}(H\nu/\mu)^2$ were assumed. The reversible permeability μ_r in (12) reduces to the initial permeability μ_i on the initial magnetization curve. Thereby, it is obvious that the domain-based Chua-type model (4c) and the mathematical-based model (10) exhibit Rayleigh's curve.[2] However, because of the fact $\mu_i > \mu_0$, the aftereffect-based model (8) does not exhibit the exact Rayleigh's curve.

B. Anhysteretic magnetization

Minor loops are generally most important for magnetic recording. Thereby, we calculated an anhysteretic magnetization process.[9] Three kinds of materials were selected for the examination. The first one is a laminated iron toroidal core, the second is a ferrite core (TDK K6A), and the third is an amorphous magnetic core (Toshiba MA1). The parameters μ, μ_r, and s of the domain-based Chua-type model were carefully measured. One of the results is shown in Fig. 2 for the ferrite. As shown in Figs. 3–5, fairly good agreement between the computed and experimental results was obtained on the overall results. However, some discrepancy in the measured and predicted minor loops may appear. This is mainly caused by the inaccuracy of the hysteresis coefficient s, because the measurement of dH/dt in (4c) is somewhat difficult.

IV. CONCLUSION

As shown above, we have examined the Chua-type magnetization models and compared with the other models. As a result, it has been revealed that the domain-based Chua-type model is capable of representing various magnetization characteristics commonly observed in practice. Further, it has been clarified that the Chua-type model can be derived by considering the aftereffect process as well as the mathematical approach. This Chua-type model is a fairly simple differential form so that it may be considered as one of the best representations for computational magnetodynamics.

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