FINITE ELEMENT SOLUTION OF PERMANENT MAGNETIC FIELD

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Abstract - Currently available CAD systems on electromagnetics use the technique of shifting known demagnetization curve of the second quadrant to the origin and introduce a suitable current carrying coil for modelling permanent magnets. This paper provides the result of a validity test obtained by simulating a rotating magnetobase. The behavior of the torque with the rotation of a permanent magnet has been calculated using this technique of shifting and compared with the experimental results. Agreement to a considerable extent confirms the validity of this technique.

INTRODUCTION

To optimize the performance of electromagnetic devices, a detailed knowledge of the field distribution is absolutely necessary. The basic governing equation of these devices are represented by Maxwell's equations. The complexity arising out of the geometry and non-linear properties of the magnetic material makes it difficult to find the exact solution from these partial differential equations, whereas numerical approximation methods, such as, finite difference method, finite element method etc., allow a convenient solution. Of the numerical analysis methods available for solving electromagnetic field equations, the finite element method has achieved preeminence. In fact, almost every magnetic analysis package now available uses the finite element method for its mathematical operation.

Modelling of magnetic material is a very important task in the magnetic field analysis. Magnetic saturation, demagnetization curves as well as other characteristics are used for modelling magnetic materials. Numerous ways of modelling magnetic material curve have been devised [1].

In case of soft magnetic materials the hysteresis is weak and, thus, the magnetic property of material is easily modelled by a piecewise linear relationship between B and H. However, in case of hard magnetic material, the hysteresis becomes strong creating difficulty in modelling the demagnetization curve. The demagnetization curve can be well approximated by a shifted B-H curve passing through the origin adjusted by a suitable current carrying coil. This relation resembles to that of Frollich-Kenelly's relation [2].

In this paper, the validity of modelling permanent magnet using the above concept has been confirmed by a concrete example of rotating magnetobase [3].

MODELLING OF PERMANENT MAGNETS

Many common devices such as motors, generators, loud speakers, telephone recievers etc., requires strong and constant magnetic field for their operations and as such, use permanent magnets. It is extremely difficult to model the actual physical representation of permanent magnet using the hysteretic phenomenon in cases of minor loops. Usually, the characteristics of hard magnetic materials is handled in a very restrictive way. The section in the second quadrant of the major hysteresis loop (Fig.1), known as demagnetization curve is used for characterization of magnetic materials.

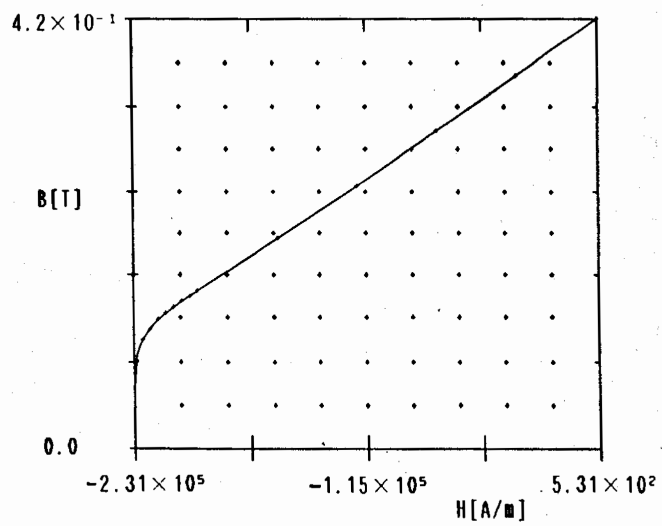

Fig. 1. Typical demagnetization curve.

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The demagnetization curve can be mathematically formulated as [1]

$$B = \frac{H+H_c}{a+b(H+H_c)}. \tag{1}$$

This resembles to Frollich Kenelly's relation, where B, H and H_c are the flux density, field intensity and coercive field, respectively. The coefficients a and b in (1) are obtained as follows.

Let, $H\rightarrow\infty$ so that, $B\rightarrow B_s$ where B_s denotes the magnetic flux density at saturation. Then,

$$B_s = \lim_{H\rightarrow\infty} \frac{H+H_c}{a+b(H+H_c)}$$

$$= \lim_{H\rightarrow\infty} \frac{1}{\frac{a}{H+H_c}+b} = \frac{1}{b},$$

$$\text{or} \quad b = \frac{1}{B_s}. \tag{2}$$

Similarly, letting $H\rightarrow 0$, we get B_r, where B_r denotes the residual or remenance flux density and

$$B_r = \lim_{H\rightarrow 0} \frac{H+H_c}{a+b(H+H_c)} = \frac{H_c}{a+bH_c},$$

$$\text{or} \quad a = H_c\left(\frac{1}{B_r}-\frac{1}{B_s}\right). \tag{3}$$

Substituting values of a and b from equations (3) and (2) respectively into (1), we get

$$B = \frac{H+H_c}{H_c\left(\frac{1}{B_r}-\frac{1}{B_s}\right)+\frac{1}{B_s}(H+H_c)} = \frac{H+H_c}{\frac{H}{B_s}+\frac{H_c}{B_r}}$$

$$= \mu(H+H_c), \tag{4}$$

where μ denotes a permeability.

Equation (4) allows us to express the demagnetization curve of first and second quadrant in a shifted curve passing through the origin and the current density due to the coercive field as shown in Fig.2.

FINITE ELEMENT FORMULATION

With the above representation the permanent magnetic material, in other words, the hard material can now be considered as consisting of a soft material and the current density equivalent to that of coercive field.

Rearranging equation (4), we obtain

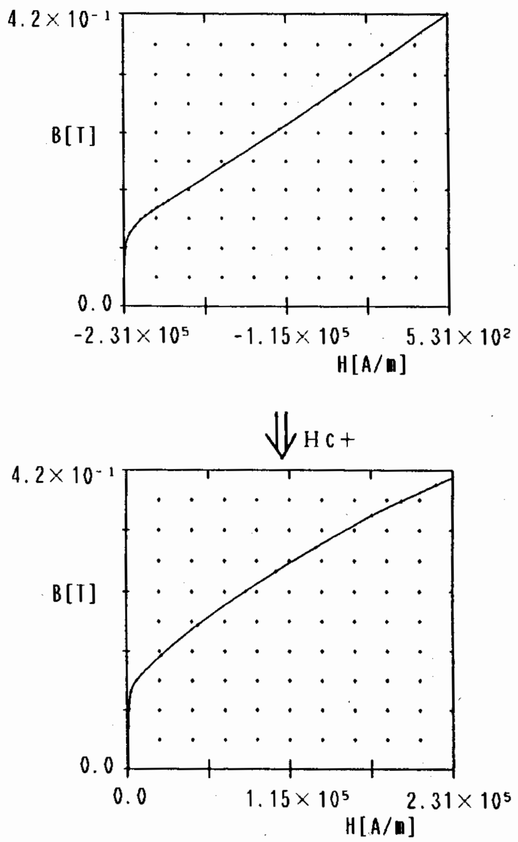


Fig. 2. Representation of demagnetization curve in the form of a shifted curve.

$$\bar{H} = \frac{1}{\mu}\bar{B}-\bar{H}_c, \tag{5}$$

and thus,

$$\nabla\times\frac{1}{\mu}\bar{B} = \nabla\times\bar{H}+\nabla\times\bar{H}_c. \tag{6}$$

Denoting a vector potential $\bar{A}$ and substituting $\bar{B}=\nabla\times\bar{A}$ in (6) we get,

$$\nabla\times\frac{1}{\mu}\nabla\times\bar{A} = \bar{J}+\bar{J}_m, \tag{7}$$

where the source current density $\bar{J}$ and equivalent current density $\bar{J}_m$ are

$$\nabla\times\bar{H} = \bar{J}, \tag{8}$$

$$\nabla\times\bar{H}_c = \bar{J}_m. \tag{9}$$

When we use a piecewise linear approximation for the permeability μ, then a functional $F(A)$ for equation (7) can be formally written as

$$F(\bar{A}) = \frac{1}{2}\int_s \frac{1}{\mu}(\nabla\times\bar{A})^2 ds - \int_s \bar{J}\cdot\bar{A}ds - \int_s \bar{J}_m\cdot\bar{A}ds. \qquad (10)$$

Characteristics of the permanent magnet can be found in the rightmost term of equation (10). Therefore, let us examine this term.

$$\int_s \bar{J}_m\cdot\bar{A}ds = \int_s \nabla\times\bar{H}_c\cdot\bar{A}ds. \qquad (11)$$

When we apply the integral to some small part of permanent magnet shown in Fig. 3, then equation (11) can be rewritten by

$$\int_s \left(-i\frac{\partial H_c}{\partial z}+k\frac{\partial H_c}{\partial x}\right)\cdot\bar{A}dxdy = \int_s k\frac{\partial H_c}{\partial x}\cdot\bar{A}dxdy, \qquad (12)$$

where we assume that the y directional coercive field H_c varies as a function of x and remains constant for z ; i and k denote the unit vectors in the direction of x and z axis respectively.

Equation (12) means that the equivalent current density $\bar{J}_m$ in equations (7) and (9) is flowing towards z direction. Let the equivalent current density $\bar{J}_m=\partial H_c/\partial x$ take a constant value in a small part of the permanent magnet, then the integration of (12) can be carried out in much the same way as the conventional one. As a result, the current term I_m caused by the equivalent current density $\bar{J}_m$ in the finite element system of equations is reduced to

$$I_m = H_c\cdot L_y, \qquad (13)$$

for the permanent magnet shown in Fig. 3, where L_y is the height of this magnet.

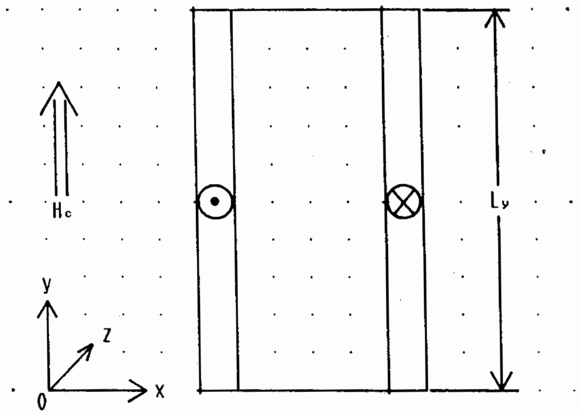
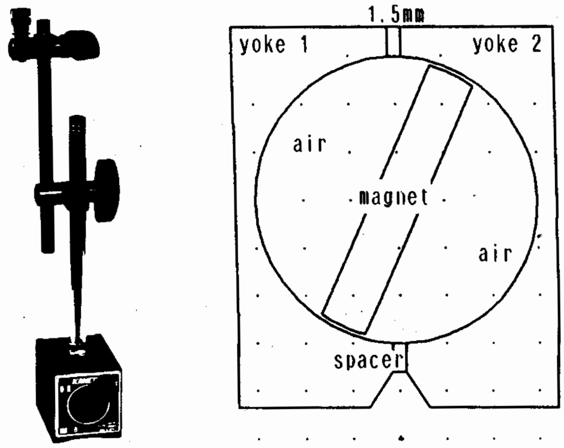


Fig. 3. Cross-Sectional view of a magnet with the current sheet on its surfaces.



(a) Photograph of rotating magnetobase.

(b) Cross-sectional view of rotating magnetobase.

Fig. 4.

ROTATING MAGNETOBASE

Magnetic base is widely used in the industries to hold the positions of rotating objects. Since its operation is depending on the permanent magnetic field, therefore, it is one of the best selections as an initial test for our problem. The magnetic base considered here (hereafter rotating magnetobase) (Fig. 4) consists of two yokes, a permanent magnet and a non-magnetic spacer positioned between the two yokes.

Flux is generated due to permanent magnet and flows through the yokes. However, by rotation of the permanent magnet through a certain angle a difference occurs in the flux in the two yokes. This causes a change in magnetic energy in the gap between the permanent magnet and the yokes, which in turn generates a force at the two poles of the magnet, tending to rotate the magnet in, respectively, the direction of rotation and the direction opposite to it.

This paper calculates the torque using Maxwell stress tensor corresponding to different angular position of the permanent magnet of the magnetic base.

MODELLING OF ROTATING MAGNETOBASE

Clearly, the problem has symmetry in the axial direction. Therefore, a two dimensional cross section is sufficient to analyze the problem quite accurately. The torque calculated at the two poles are based on Maxwell stress tensor method which is described as follows.

An arbitrary surface enclosing the object for which the torque needs to be calculated is chosen. The torque τ at a point at a distance r is calculated by

$$\tau = \int_s \mu_0\left[(\bar{H}\cdot\bar{n})(\bar{r}\times\bar{H})-\frac{1}{2}\bar{H}^2(\bar{r}\times\bar{n})\right] ds, \qquad (14)$$

where n is the normal vector to the surface S, H the magnetic field and μ_0 the permeability of free space

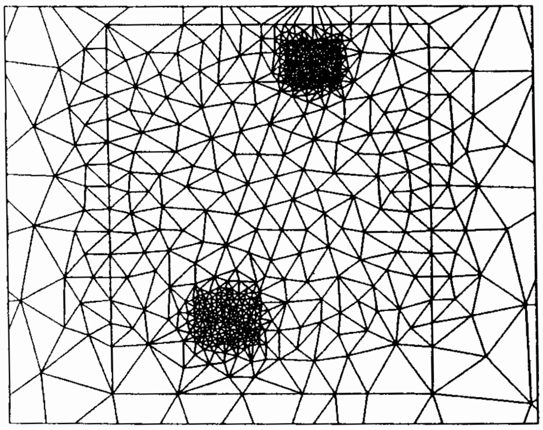


Fig. 5. Partial view of the mesh of rotating magnetobase.

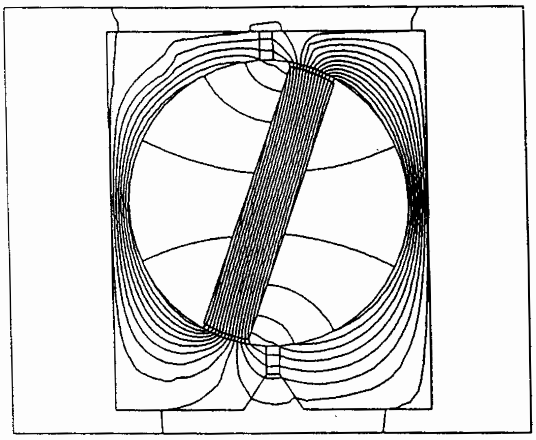


Fig. 6. Flux distribution of rotating magnetobase.

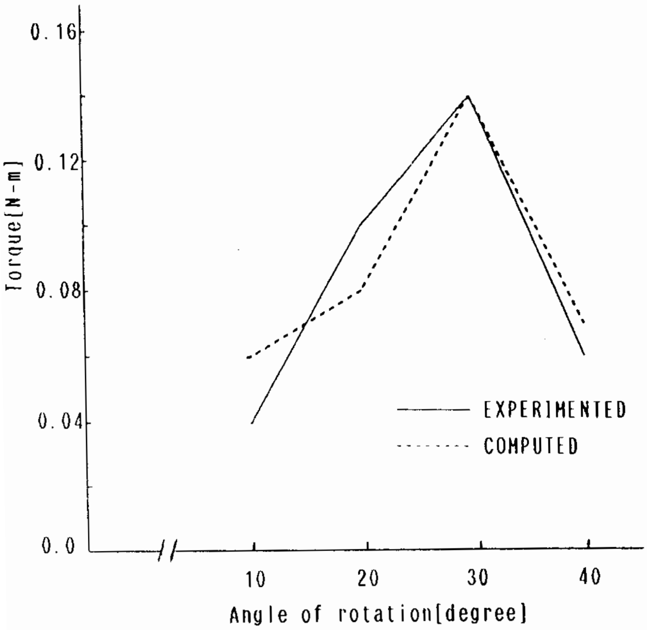


Fig. 7. Torque v.s angle of rotation.

[4]. A mesh of 1000 triangles (Fig. 5) was formed to analyze the problem. Fig. 6 shows the flux distribution corresponding to 10 degree of angle of rotation. Fig. 7 is a plot of computed (on the basis of shifting procedure) and experimented results of torque at different angles of rotation corresponding to Ferrite. It can be clearly seen that the computed result agree with the experimental result.

CONCLUSION

Modelling of the permanent magnet in the finite element analysis is not a trivial task. Complexity arising out of the hysteretic phenomenon, in particular, are difficult to handle. However, the procedure of shifting known demagnetization curve to the origin and adjustment by a suitable current carrying coil provides good approximation of the actual physical phenomenon. In this paper a CAD package [5], which uses this technique, has been used for validity test and is observed that this technique can well approximate the actual behavior of permanent magnet.

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