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Finite Element Solution of Open Boundary Eddy Current Problems

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ABSTRACT

Previously, we have proposed a new method which is based on the strategic dual image (SDI) forcing an open boundary to close for the finite element solution of open boundary problems (Saito *et al.*, 1987). The theoretical background of this SDI method is based on the generalization of traditional electrical image method so that the finite element as well as finite difference solution of open field problems can be obtained in an extremely simple manner (Saito *et al.*, 1988). Furthermore, the finite element solution of saturable magnetic field problems has been successfully obtained by combining our SDI and magnetization vector **M** (Saito *et al.*, 1988).

In this paper, we apply this SDI method to the dynamic magnetic field problems. As a result, it is reveald that the finite element solution of open boundary eddy current problems can be obtained in an extremely simple manner.

KEYWORDS

Eddy current; Open boundary; Image.

INTRODUCTION

Unbounded open field problems arise in the analysis of electric and magnetic fields in engineering applications. Typical examples of these are the determination of dielectric stresses in power line insulators, effect of switching surges on power devices, magnetic field distribution in accelerator magnets, magnetic printer heads, contactors and actuators. The finite element and difference methods are being widely used to solve the various engineering field problems. However, because of their essential feature, it is difficult to obtain the solution of the unbounded field problems. To overcome this difficulty, various means have been proposed in the literature such as ballooning, infinitesimal scaling and hybrid boundary integral techniques. So far ballooning has been used for 2D problems (Silvester *et al.*, 1977). Infinitesimal scaling is useful but requires the solution of a nonlinear matrix equation which is time consuming (Crowley *et al.*, 1985). The hybrid integral method leads to an unsymmetric matrix and requires the long solution time and large computer storage capacity (Salon *et al.*, 1982). To remove these deficiencies of the existing methods, we have previously proposed a new method which is based on the method of electrical image. This new method is called the strategic dual image (SDI) method. The theoretical background of this SDI method is based on the generalization of traditional electrical image method so that the finite element as well as finite difference solutions of open field problems can be obtained in an extremely simple manner (Saito *et al.*, 1987, 1988). Even if the saturation of magnetic materials is taken into account, the finite element solution has been successfully obtained by our SDI method (Saito *et al.*, 1988).

In this paper, we apply this SDI method to the dynamic magnetic field problems. As a result, it is revealed that the finite element solution of open boundary eddy current problems can be obtained in an extremely simple manner.

THE STRATEGIC DUAL IMAGE METHOD

Basic field equations

Most of the magnetodynamic field problems can be reduced to solve a following governing equation:

$$\nabla \times \left(\frac{1}{\mu} \right) \nabla \times \mathrm{A} + \sigma \frac{\partial \mathrm{A}}{\partial t} = \mathrm{J}_s , \qquad (1)$$

where A, J_s, μ and σ are respectively the vector potential, source current density, permeability and electric conductivity. The vector potential A is related with the flux density B by

$$\nabla \times \mathrm{A} = \mathrm{B} , \qquad (2)$$

so that, by considering a relationship between the electric field intensity E and flux density B, it is possible to derive a following relation:

$$\mathrm{E} = -\nabla \phi - \frac{\partial \mathrm{A}}{\partial t} , \qquad (3)$$

where ϕ denotes an arbitrary scalar potential. The electric field intensity E is related with the current density J by

$$\mathrm{J} = \sigma \mathrm{E} . \qquad (4)$$

By means of Eqs.(3) and (4), it is possible to show that the source current density J_s in Eq.(1) has been assumed to

$$\mathrm{J}_s = -\sigma \nabla \phi . \qquad (5)$$

Thereby, the eddy current density J_e is given by

$$\mathrm{J}_e = -\sigma \frac{\partial \mathrm{A}}{\partial t} . \qquad (6)$$

<u>Assumptions</u>

At first, the net magnetic field source is assumed to be zero in the problem region. Second, as shown in Fig.1, it is assumed that the problem region is enclosed by a boundary located at an infinitely long distance from the problem region. This assumption means that the boundary conditions at this infinite boundary are

$$\mathrm{B}_n = 0 , \qquad (7)$$

$$\mathrm{H}_t = 0 , \qquad (8)$$

where the flux density B_n and field intensity H_t are respectively the normal and tangential components to the infinite boundary as illustrated in Fig.1. Further, Eqs.(7) and (8) suggest that the open field is composed of the divergence field B_n and rotational field H_t . Third, it is assumed that the magnetic field source may be regarded as a rotational field source, i.e., current i, or a divergence field source, i.e., magnetic charge m. Finally, we assume that there is no conducting media spreading to the infinitely long distance. This assumption means that the eddy current is always confined in a finite conducting media.

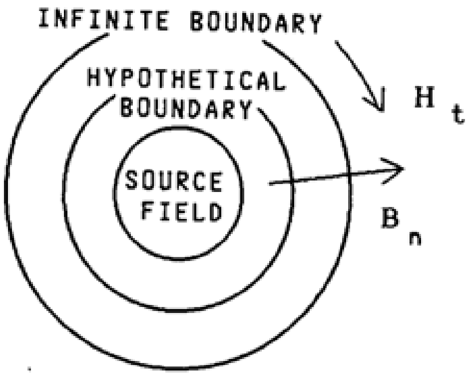

Fig.1. Modeling of open field problem.

Strategic dual images

Let consider one of the current i in the problem region. When we impose an image current $-(d/a)i$ at a position shown in Fig.2 (a), then the normal component of flux density B_n becomes zero at an arbitrary point on the spherical surface enclosing the source current i. This means that the normal component of flux density B_n to the spherical surface is suppressed by the image current $-(d/a)i$ so that the remainig field is the tangential field intensity H_t to the spherical surface. This field intensity H_t becomes zero at the infinite boundary. The magnitude of image $-(d/a)i$ depends on a position of field source i in the sphere so that the following condition

$$a\sum_{p=1}^{q}(i_p/r_p)=0, \tag{9}$$

must be satisfied to reduce the zero net image. In Eq.(9), a is a radius of sphere; r_p $(=a^2/d_p)$ is a distance from the center of sphere to the current i_p; and q is a number of sources.

Let consider one of the magnetic charges m in the problem region. When we impose an image $-(d/a)m$ at a position shown in Fig.2(b), then the tangential component of field intensity H_t becomes zero at an arbitrary point on the spherical surface enclosing the field source m. This means that the tangential component of field intensity H_t to the spherical surface is suppressed by the image $-(d/a)m$ so that the remaining field in the sphere is the normal component of flux density B_n, and this becomes zero at the infinite boundary. Similar to those of Eq.(9), the following condition

$$a\sum_{p=1}^{q}(m_p/r_p)=0, \tag{10}$$

must be satisfied to reduce the zero net image.

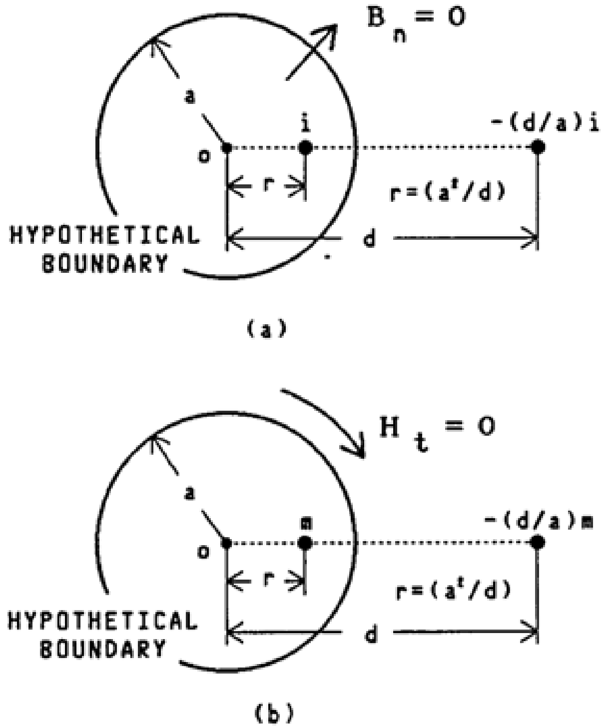

Fig.2. Strategic dual images. (a) The rotational field source image $-(d/a)i$ and spherical hypothetical boundary. The normal components of flux density B_n at the spherical surface becomes zero. (b) The divergence field source image $-(d/a)m$ and spherical hypothetical boundary. The tangential components of field intensity H_t at the spherical surface becomes zero.

Open field

The field intensity H_t which satisfies condition (8) is obtained by imposing the rotational field source image. Also, the flux density B_n which satisfies condition (7) is obtained by imposing the divergence field source image. Therefore, the open field can be obtained by a summation of these two fields, that is

$$\text{open field} = (1/2)(\text{rotational field} + \text{divergence field}), \qquad (11)$$

where coefficient (1/2) is come from the two field sources (rotational and divergence field sources).

Implementation

In Eq.(2), the magnetic flux density **B** is represented by the curl of vector potential **A** so that the vectors representing the rotational and divergence field components are respectively calculated by setting the boundary conditions $A=0$ and $\partial A/\partial n=0$ at the spherical surface of the hypothetical boundary. Furthermore, because of Eq.(9), $A=0$ must be satisfied at the center of spherical surface. After governing equation (1) is discretized by the conventional finite element method imposing the zero or symmetrical boundary conditions to the hypothetical boundary, the distributed field source is substantially concentrated at the node points so that the condition (9) can be iteratively satisfied for the distributed field source.

Let assume a discretized system of equations using the symmetrical boundary condition as

$$\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} D & 0 \\ 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} F_s \\ 0 \end{bmatrix}, \qquad (12)$$

where X_1 is a sub-vector on the inside region; X_2 is a sub-vector on the hypothetical boundary; F_s is an input vector caused by J_s in Eq.(1); sub-matrices C_{11}, C_{12}, C_{21}, C_{22} are correspondingly defined to the sub-vectors X_1, X_2; and D is a sub-matrix caused from the time differential term of Eq.(1).

According to Eq.(11), the SDI solution vector X_2' on the hypothetical boundary is given by

$$X_2' = \frac{1}{2} X_2$$
$$= -\frac{1}{2} [C_{22} - C_{21}C_{11}^{-1}C_{12}]^{-1} C_{21}C_{11}^{-1}$$
$$\times [F_s - D(\frac{d}{dt}) X_1'] , \qquad (13)$$

because the sub-vector X_2' of zero boundary system is always zero vector; and X_1' denotes the SDI solution vector on the inside region. Rearrangiment of Eq.(13) yields

$$[2C_{22} - C_{21}C_{11}^{-1}C_{12}] X_2' + C_{21}X_1' + C_{21}C_{11}^{-1} [F_s$$
$$- C_{11}X_1' - C_{12}X_2' - D(\frac{d}{dt}) X_1'] = 0. \qquad (14)$$

In Eq.(14), obviously following relationships are established:

$$C_{11}X_1' + C_{12}X_2' + D(\frac{d}{dt}) X_1' = F_s , \qquad (15)$$

$$C_{21}X_1' + [2C_{22} - C_{21}C_{11}^{-1}C_{12}] X_2' = 0. \qquad (16)$$

By means of Eqs.(15) and (16), it is possible to obtain

$$CX_1' + D(\frac{d}{dt}) X_1' = F_s , \qquad (17)$$

where

$$C = C_{11} - C_{12} [2C_{22} - C_{21}C_{11}^{-1}C_{12}]^{-1} C_{21}, \qquad (18)$$

and

$$X_2' = - [2C_{22} - C_{21}C_{11}^{-1}C_{12}]^{-1} C_{21}X_1'. \qquad (19)$$

Equation (17) is discretized in time by the backward difference method.

An example

In the present paper, the strategic dual image method has been formulated in three dimensions. However, because of its simplicity, the two dimensional magnetodynamic field problems are far more preferable as the initial test examples.

As shown in Fig.3, we selected a simplified model of induction heating cooker as an example. The discretization of this example was carried out by means of the standard first order triangular finite elements. Various constants used in the computations are listed in Table I. Figure 4 shows the step response of this model. The solutions were obtained using the hypothetical boundaries with different radii. Nevertheless, the results in Fig.4 obviously suggests that our SDI method gives an unique solution. Further, Fig.5 shows the eddy current distributions caused by a step input current.

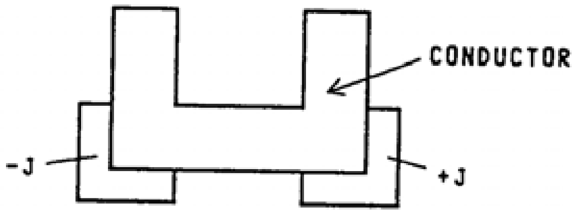

Fig.3. A simplified model of induction heating cooker.

TABLE I. Various constants used in the computations.

Number of elements		3m:248, 4m:432, 5m:664
Source Current Density	J_s	1e+4[A/m²]
Step-width in time	Δt	0.01[s]
Conductivity	σ	1e+7[s/m]

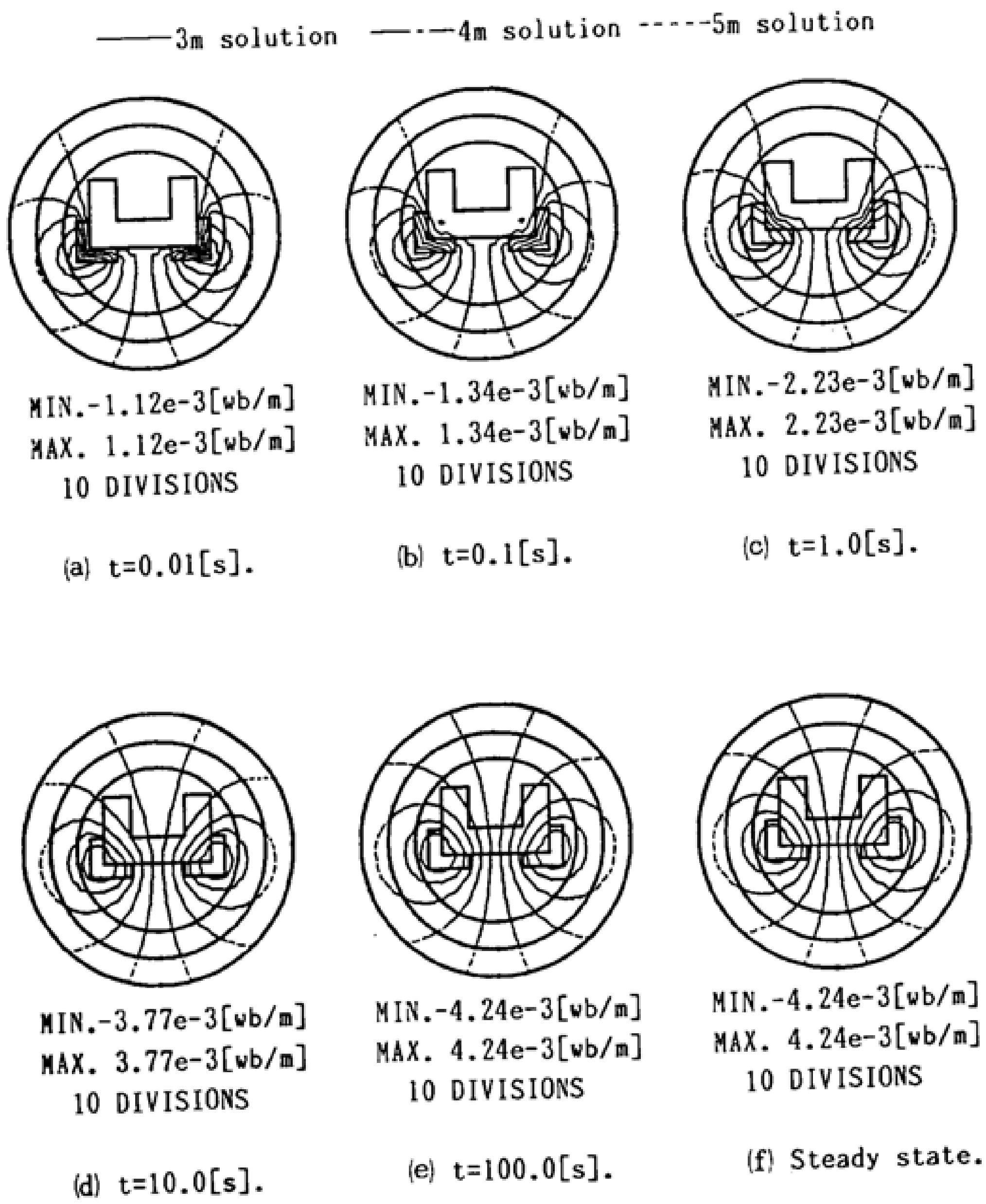

Fig.4. Step responce of the induction heating cooker.

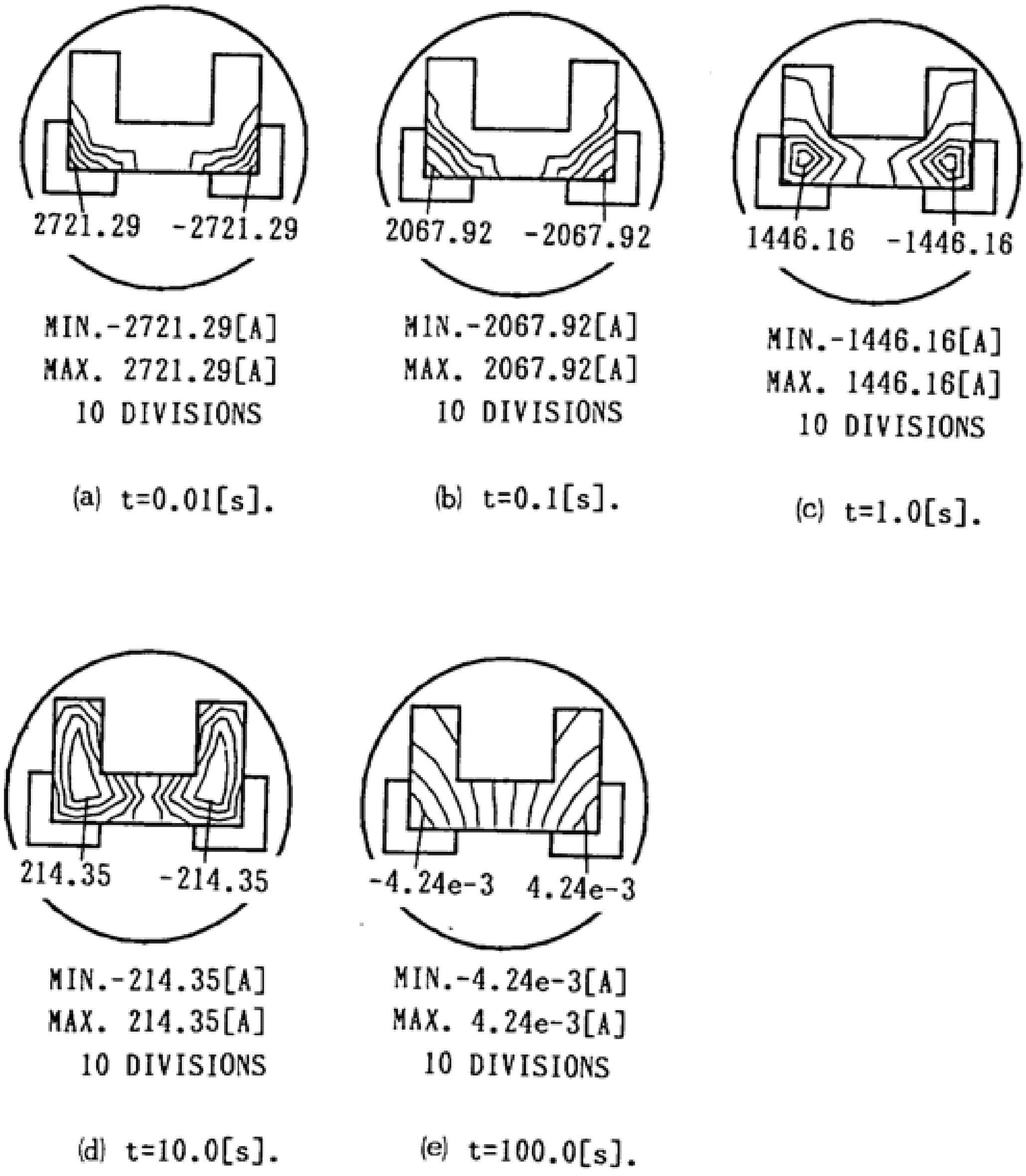

Fig.5. Eddy current distributions caused by a step input current.

CONCLUTION

As shown above, we have revealed that our strategic dual image method is still effective procedure for obtaining the finite element solution of open boundary eddy current problems in an extremely simple manner. Fundamentally, our strategic dual image method is an analytical mean for the open field problems so that it is applicable to the other discretization method (e.g. finite difference method) for obtaining the open field solution.

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