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**APPLICATIONS OF
ELECTROMAGNETIC PHENOMENA IN
ELECTRICAL AND MECHANICAL SYSTEMS**

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ABSTRACT: This paper proposes a method of electromagnetic field distribution analysis by means of image processing. The key idea of our image processing methodology is that each of the pixels representing an image is regarded as a kind of potential in vector fields. Based on the vector calculus in classical physics, any static and dynamic image can be obtained by the solutions of Poisson- and Helmholtz- type partial differential equations, respectively. This makes it possible to handle any image as an analytical and continuous quantity. As an example of the image Helmholtz equation, we have evaluated the magnetization process of a grain-oriented silicon steel sheet from its magnetic domain images obtained by a scanning electron microscope. The magnetization curve was computed from the domain images obtained by the solutions of image Helmholtz equations. The state transition matrices derived from the Helmholtz equation turned out to be one of the evaluation methodologies linking magnetization process and magnetic domain behavior.

1 INTRODUCTION

Numerous visualized images can be currently obtained by various methods, e.g. Charge Coupled Device (CCD), Magnetic Resonance Image (MRI), etc. Scanning Electron Microscope (SEM) also provides images representing the electron array of materials and is widely used for evaluating properties of magnetic materials [1-3]. Observation of magnetic domains from such images is one of the important standards for evaluating the phenomena in magnetic materials, such as magnetic saturation and magnetic hysteresis. In most cases, such tasks require a high degree of expertise.

Recently, an image processing methodology based on classical physics has been proposed. Regarding an image as a scalar or a vector potential field, the conventional vector operations applied to the image become useful tools for image processing as well as for computer graphics. As a result of that strategy, any digital

images can be represented by partial differential equations. Particularly, a solution of the image Helmholtz equation gives the dynamic images, e.g., computer graphics animation and moving objects captured by video camera. This approach makes it possible to handle the dynamic images, which are composed of several static images such as frames, as continuous quantity. Further, a characteristic value of the differential equation corresponds to a physical constant so that our approach has capability for identifying the characteristics of physical systems from visualized information.

This paper presents a method of evaluation of the magnetization process of a grain-oriented silicon steel sheet by means of the image Helmholtz equation. We have estimated the characteristic values from the magnetic domain images obtained by SEM, and computed a magnetization curve from each of the SEM images. As a result, we have succeeded in confirming the conventional magnetic domain theory.

2 IMAGE PROCESSING BY HELMHOLTZ EQUATION

2.1 Image Helmholtz equation

Many physical dynamic systems are governed by the Helmholtz type of equations. Assuming that an image is a scalar field U , any dynamic images with arbitrary resolution are also given as a solution of the Helmholtz equation:

$$\nabla^2 U + \varepsilon \frac{\partial}{\partial \alpha} U = -\sigma, \quad (1)$$

where ε , α and σ denote a moving speed parameter, a transition variable and an image source density, respectively. The first and the second term on the left in Eq.(1) represent the spatial expanse and transition of image to the variable α , respectively. In case of movies, the variable α replaces the time t . The first term on the left in Eq.(1) represents a static image and the image source density σ is given by Laplacian operation to a final image so that the final image U_{Final} is obtained as a solution of

$$\nabla^2 U_{Final} = -\sigma. \quad (2)$$

This means that the governing equation of static images is the Poisson equation [4].

2.2 Solution of the image Helmholtz equation

The modal analysis of Eq.(1) gives a general solution:

$$U(\alpha) = \exp(-\Lambda\alpha)(U_{Start} - U_{Final}) + U_{Final}, \quad (3)$$

where U_{Start} and Λ are an initial image and a state transition matrix, respectively [5]. The values ε and σ in Eq.(2) are reduced into the matrices Λ and U_{Final} . It should be noted that the matrix Λ is unknown, since the value ε in Eq. (1) is not given. The state transition matrix needs to be determined from the given images.

2.3 State transition matrix

The state transition matrix Λ is a key to generate the dynamic image because it corresponds to the characteristic values, which

are the reciprocal of time constants in various engineering applications. In most cases, the state transition matrix is given and derived from various conditions as well as from physical constants of the system. In such a case, we are able to obtain its solution uniquely. In our case, however, we have to determine it from several images representing the results. The process discovering the conditions from the results is an inverse problem.

If we define the image $U_{\Delta\alpha}$ when the variable α takes an arbitrary value $\Delta\alpha$ from the interval between the initial and final image, then it is possible to determine the state transition matrix Λ by modifying Eq.(3).

$$\Lambda = -\frac{1}{\Delta\alpha} \ln \left(\frac{U_{\Delta\alpha} - U_{Final}}{U_{Start} - U_{Final}} \right). \quad (4)$$

Therefore, it is possible to analytically generate the dynamic image by substituting Eq.(4) into Eq.(3).

3 ANALYSIS OF MAGNETIC DOMAIN MOVEMENT

3.1 Magnetic domain images by SEM

Fig.1 shows the magnetic domain images of a grain-oriented silicon steel sheet by SEM at the magnetic flux densities $B=0, 1.5, 1.7$ and 1.8 T when the magnetic field strengths are equal to $H=0, 20, 40$ and 140 A/m, respectively. Each of the images has resolution of 256 by 256 pixels.

3.2 State transition matrices and their physical meaning

Let us consider the magnetization process using the image Helmholtz equations. In this case, the flux density B is assumed to be an average of contrast of the image because Fig.1 corresponds to $(B_x, B_y, 0) = \text{grad } A_z \times \mathbf{n}_z$. Moreover, the transition variable α in Eq.(1) is the magnetic field H so that Eqs.(1) and (3) are respectively rewritten by:

$$\nabla^2 U + \varepsilon \frac{\partial}{\partial H} U = -\sigma, \quad (5)$$

$$U(H) = \exp(-\Lambda H)(U_{Start} - U_{Final}) + U_{Final}. \quad (6)$$

In order to apply these equations, it is necessary

to determine the state transition matrix Λ from the given images. In the present example, we have determined each of the durations, i.e., corresponding to the field H changes from 0 to 20, 20 to 40 and 40 to 140 A/m, of the matrices using the three images, by following modified equation:

$$\Lambda_i = -\frac{1}{\Delta H} \ln \left(\frac{U_{i+1} - U_{i+2}}{U_i - U_{i+2}} \right), \quad i = 1, 2, 3. \quad (7)$$

The images U_i and U_{i+2} in Eq.(7) correspond to the initial and final image, respectively, and U_{i+1} corresponds to the image between U_i and U_{i+2} . For example, when calculating the matrix in duration from $H=0$ to $H=20$ A/m, U_i , U_{i+1} and U_{i+2} become the images corresponding to $H=0$, 20 and 40 A/m, respectively. We have also a magnetic domain image at $B=1.9$ and $H=240$ (not shown) so it is possible to calculate the value of Λ_3 .

Figs 2(a)-(c) show the real and imaginary parts of state transition matrices at the durations 1, 2 and 3. The matrix shown in Fig.2(a) has the distinct values along the domain walls. These are the irreversible boundary displacements. Thus, we have a result exactly corresponding to the classical domain theory.

The majority of elements in matrix shown in Fig.2(b) are distributed at random along the magnetic walls. However, some parts are equal to zero. Both the movement of domain walls and rotation of magnetization within the domains cause this magnetization process. This process happens when the mode of magnetization changes and may be both irreversible and reversible and characterized by both zero and small characteristic values comparing with the duration $0 \leq H \leq 20$ A/m shown in Fig.2(a).

Finally, in Fig.2(c), all elements in the matrix Λ_3 are governed with one constant value without imaginary part. According to the conventional domain theory, this process is reversible because the majority of magnetization states represent the same direction. Also, it is visible that this is a non-linear system because the state transition matrices are different at each of the durations.

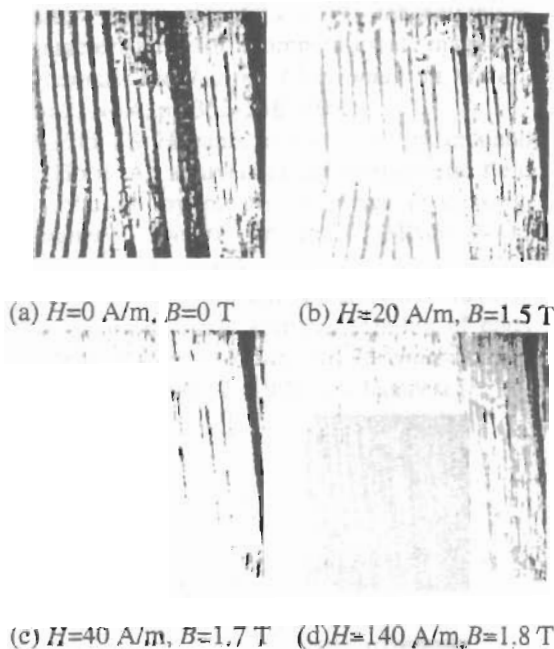


Figure 1. Magnetic domain images by SEM

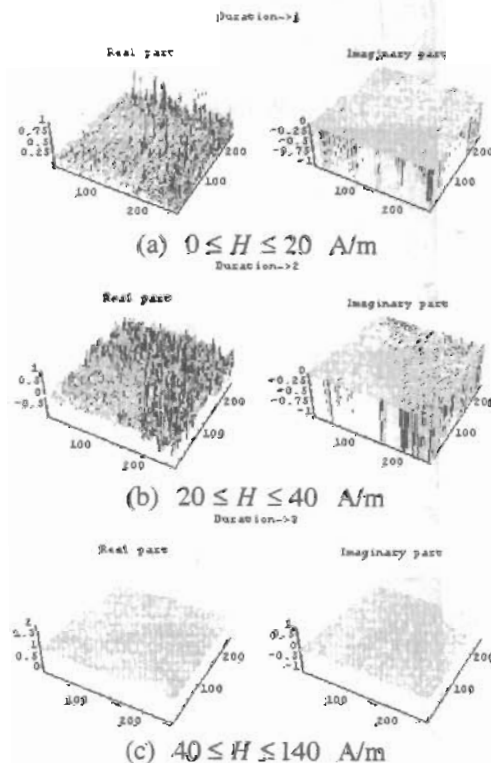


Figure 2 State transition matrices of the image Helmholtz equation. The left and right columns are the real and imaginary parts of matrices, respectively.

Let us consider the physical meaning of state transition matrices shown in Fig.2. For a ferromagnetic material, the relationship between the magnetic field and flux density can be represented by:

$$\frac{1}{\mu} \mathbf{B} + \frac{1}{\varphi} \frac{\partial \mathbf{B}}{\partial (\mathbf{H}_{ext} - \mathbf{H}_c)} = \mathbf{H}_{ext} - \mathbf{H}_c, \quad (8)$$

where \mathbf{H}_{ext} and \mathbf{H}_c represent the external field and coercive fields, respectively. Moreover, φ denotes the Preisach density function [1-3]. It's reciprocal is represented by the state transition matrices shown in Fig.2; compare Eqs.(5) and (8).

3.3 Magnetization curve reconstruction

From the results of Fig.2, we are able to observe the magnetization process as an animation. Fig.3 shows each frame of the animation generated by means of Eq.(6) (right column) together with the magnetization curves (left column). It is obvious that we have succeeded in estimating the magnetization process from only four images.

4 CONCLUSIONS

We have proposed a method of image processing based on classical physics, and also a new approach using visualized image. In this paper, we have applied our methodology to evaluate the property of a grain-oriented silicon steel sheet. Observation of the state transition matrices confirms the conventional magnetic domain theory. We anticipate that our methodology can become an intellectual tool for discovering the rules of systems.

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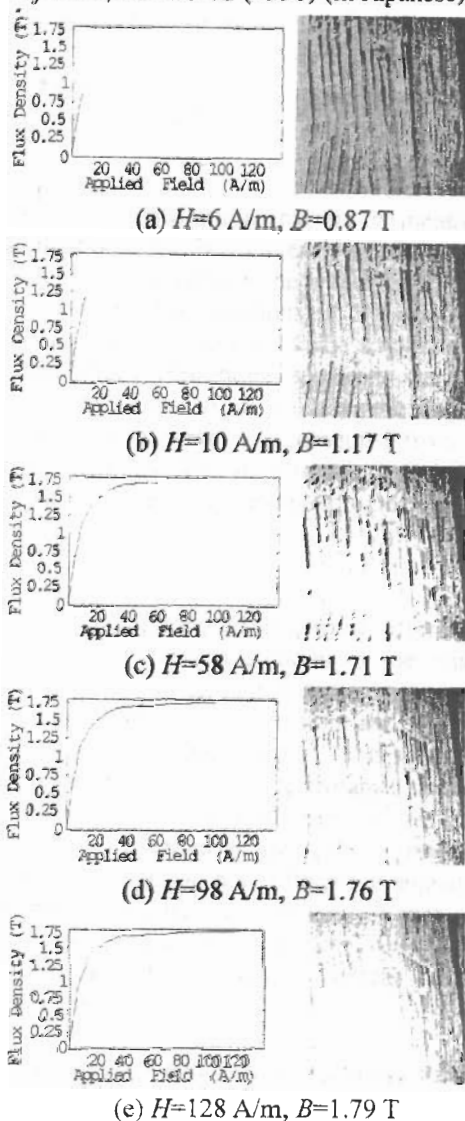


Figure 3 Magnetization process estimation by means of image Helmholtz equations. Left column: magnetization curves. Right column: generated images.