Proceedings of FEDSM'98
1998 ASME Fluids Engineering Division Summer Meeting
June 21-5, 1998, Washington, DC

FEDSM98-4828

WAVELETS STRATEGY FOR VORTEX FLOW WITH VELOCITY ON FLUID DYNAMICS

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ABSTRACT

We propose a new methodology for the noisy vortex flow velocity vectors based on wavelet transform. At first, raw vortex flow velocity vectors are represented by the vector and the scalar potentials. Applying the wavelet transform to each of the potentials, it is possible to extract the most dominant noise free vortex flow vectors.

1. INTRODUCTION

Recently, discrete wavelet transform has been widely used for the waveform analysis, image data processing(Saito,1996), also applied to solving for a linear system of equations hida,1997). Further, it has been reported (Matsuyama et al.,1997a,Matsuyama et al.,1997b) that the wavelet transform can be efficiently applied to vector data reducing the noise. Also, visualization of two dimensional vortex flow has been carried out by the wavelet auto correlation technique(Li and Nozaki,1995,Li and Nozaki,1996).

In this present paper, we propose a new methodology in order to remove the noise vector from vortex flow with velocity on fluid dynamics. Our methodology consists of two steps. The first is to evaluating the vector and scalar potentials from two dimensional noisy raw vortex flow. This step is based on the Helmholtz's theorem, but it is essential to solve for an ill posed system of linear equations. We employ the minimum norm method whose solution takes a minimum norm constraint. This minimum norm constraint always gives a unique solution vector even if the system of equations is ill posed. At second step, we apply the discrete wavelet transform to each of vector and scalar potentials. Utilizing the data compression ability of the discrete wavelet transform, noise components included in the potentials are reduced. Rotation of vector potential and gradient of scalar potential yield the rotational and divergence components of the refined vortex flow, respectively.

Thus, we have succeeded in obtaining the refined vortex flow from the raw noisy vortex flow with velocity. Simple simulation verified our methodology.

2. NOISE REDUCTION OF VORTEX FLOW WITH VELOCITY

1) Potential evaluation

According to Helmholtz's theorem, fluid velocity **U** consists of the rotational component represented by vector potential **V** and the divergent component represented by the scalar potential ϕ .,i.e.

$$\mathbf{U} = rot\mathbf{V} - grad\phi. \tag{1}$$

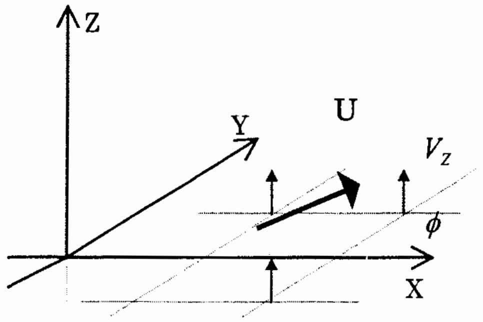


Fig.1 Vector **U**

Let us consider a two dimensional vector **U** shown in Fig.1, then our first problem is to evaluate the vector potential V_z and scalar ϕ. Because of the two dimensional problem, rotational component of the velocity vector **U** should be represented by

$$\nabla \times \mathrm{V} = \begin{bmatrix} \mathbf{i}_x & \mathbf{i}_y & \mathbf{i}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 0 & 0 & V_z \end{bmatrix}, \qquad (2)$$
$$= \mathbf{i}_x \frac{\partial V_z}{\partial y} - \mathbf{i}_y \frac{\partial V_z}{\partial x}.$$

Similarly, divergence component of the velocity vector **U** should be represented by

$$\nabla\phi = \mathbf{i}_x \frac{\partial \phi}{\partial x} + \mathbf{i}_y \frac{\partial \phi}{\partial y}. \qquad (3)$$

Combination of the Eqs. (2) and (3) yields

$$= \nabla \times \mathbf{V} - \nabla\phi$$
$$= \left(\frac{\partial V_z}{\partial y} - \frac{\partial \phi}{\partial x}\right)\mathbf{i}_x - \left(\frac{\partial V_z}{\partial x} + \frac{\partial \phi}{\partial y}\right)\mathbf{i}_y. \qquad (4)$$

Discretization of (4) by central finite difference, it is possible to derive a following system of equations, viz.,

$$\begin{aligned} \mathbf{U} &= \mathbf{U}_\mathrm{V} + \mathbf{U}_\mathrm{S}, \\ &= D_\mathrm{V}\mathbf{V} + D_S\mathbf{\Phi}, \\ &= \begin{pmatrix} D_\mathrm{V} & D_S \end{pmatrix} \begin{bmatrix} \mathbf{V} \\ \mathbf{\Phi} \end{bmatrix}, \\ &= A\mathbf{f}, \end{aligned} \qquad (5)$$

where the velocity vector **U** is a sum of rotational $\mathbf{U}_\mathrm{V}(= \nabla \times \mathbf{V})$ and a divergent $\mathbf{U}_\mathrm{S}\,(= -\nabla\Phi\,)$components. D_V and D_S are the rotational and gradient operators, respectively.

As shown in Fig. 1, a number of unknown potentials is always larger than those of given velocity vector **U**, so that Eq.(5) becomes an ill posed system of equations. In order to evaluate a unique solution vector **f** from Eq.(5), we apply the minimum norm method which is given by

$$\mathbf{f} = A^\mathrm{T}\left(AA^\mathrm{T}\right)^{-1}\mathbf{U}. \qquad (6)$$

The solution vector **f** by Eq.(6) automatically satisfies the following gauge constraints:

$$\sum_{i=1}^{m} V_i = 0, \qquad \sum_{i=1}^{m} \phi_i = 0, \qquad (7)$$

where m refers to a number of nodal points in Fig.1.

Thus, we have obtained the solution vector **f** uniquely. As a concrete example, Fig. 2 shows a noisy velocity distribution of vortex flow. The vector and scalar potential distributions for this model are shown in Figs. 3(a) and 3(b), respectively.

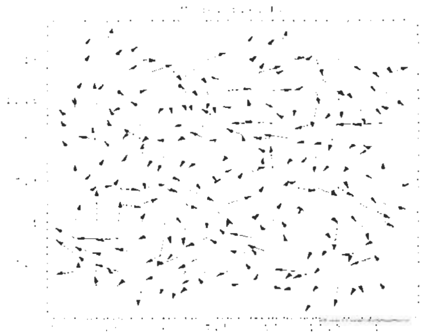
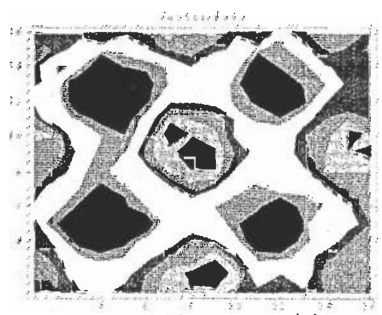
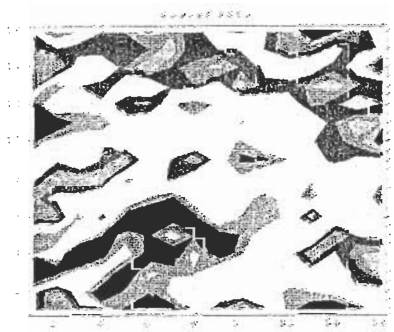


Fig.2 Model noisy velocity distribution with vortex flow.



(a) Vector potential



(b) Scalar potential

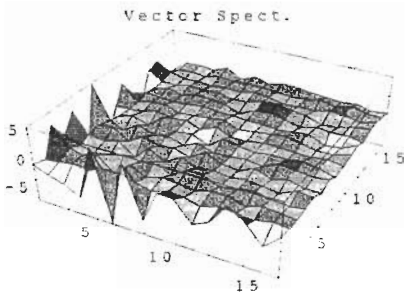
Fig.3 Vector and scalar potential distributions evaluated from noisy velocity distribution of vortex flow.

2) Wavelet transform

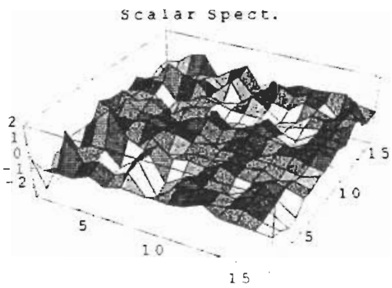
Denoting W_m as a wavelet transform matrix, the vector **V** and Φ in Eq.(5) are respectively transformed by

$$V' = W_m V W_m^T,$$
$$\Phi' = W_m \phi W_m^T, \qquad (8)$$

where V' and Φ' are the wavelet spectrum vectors. Also a superscript T refers to the transpose of matrix. The wavelet spectra of the vector and scalar potentials in Fig. 3 are shown in Figs. 4(a) and 4(b), respectively.



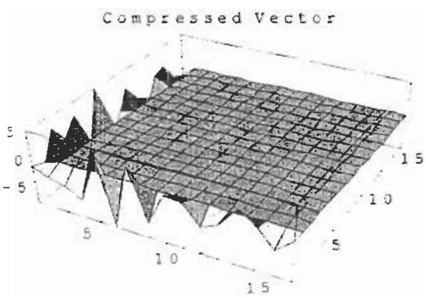
(a)Vector potential



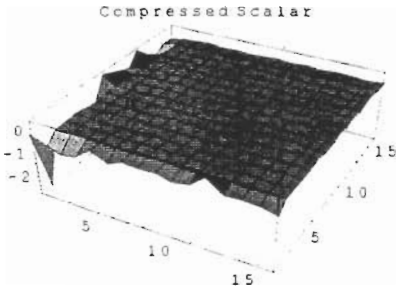
(b) Scalar potential

Fig.4 The wavelet spectra of vector(a) and scalar(b) potentials.

As shown in Fig.5, the noise contained in the vortex flow in Fig.1 is pushed through by setting the elements to be zero excepting the 1st row and column elements in Fig.4.



(a) Vector V''



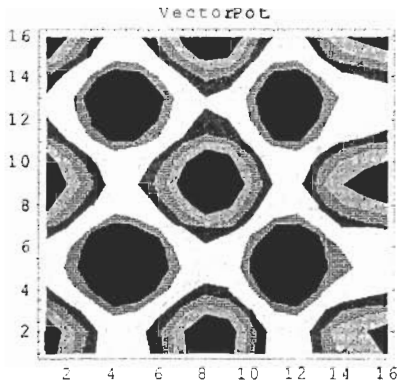
(b) Scalar potentials Φ''

Fig.5 The modified wavelet spectra setting the elements to be zero excepting the 1st row and column elements in Fig.4.

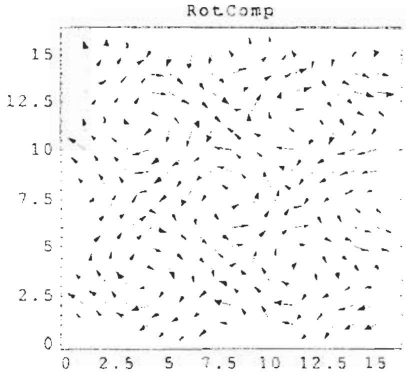
Inverse wavelet transform for the modified wavelet spectra V'' and Φ'' are carried out by

$$V_{recovered} = W_m^T V'' W_m, \qquad (9)$$
$$\phi_{recovered} = W_m^T \Phi'' W_m.$$

Fig. 6 shows the vector potential $V_{ecovered}$ and its vector distributions. Also, Fig. 7 shows the scalar potential $\Phi_{recovered}$ and its vector distributions.

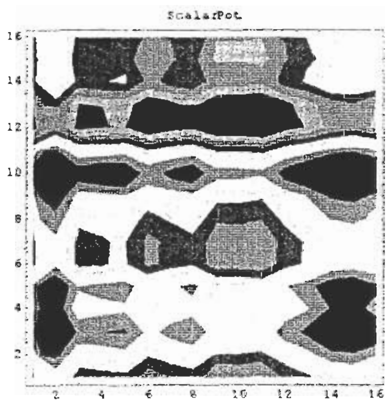


(a)

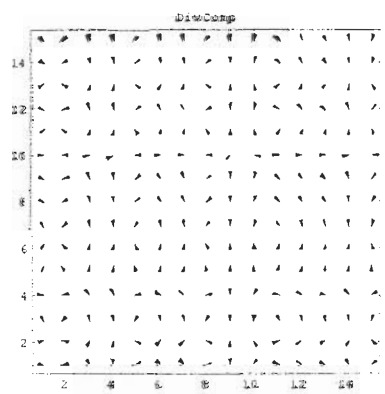


(b)

Fig. 6. Recovered vector potential $V_{recovered}$ and its vector distributions.



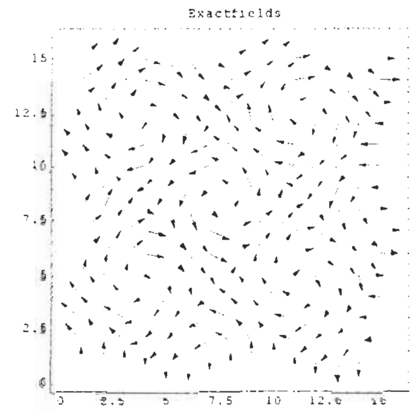
(a)



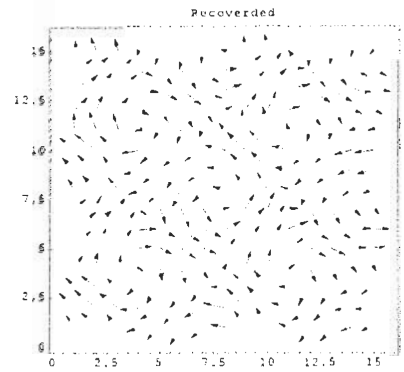
(b)

Fig.7. Scalar potential $\Phi_{recovered}$ and its vector distributions.

According to Eq.(4), summation of the vectors in Figs. 6(b) and 7(b) yields a noise free vortex flow as shown in Fig.8.



(a) Exact



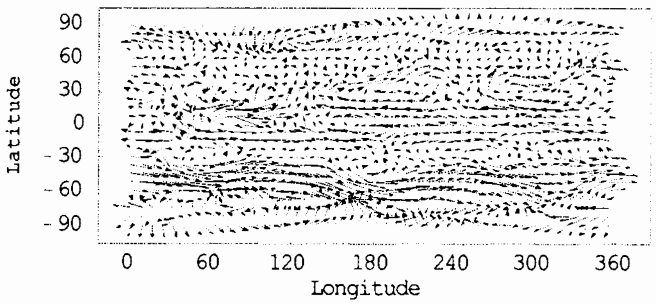
(b) Refined

Fig.8. Comparison of the exact and refined vortex flow with velocity.

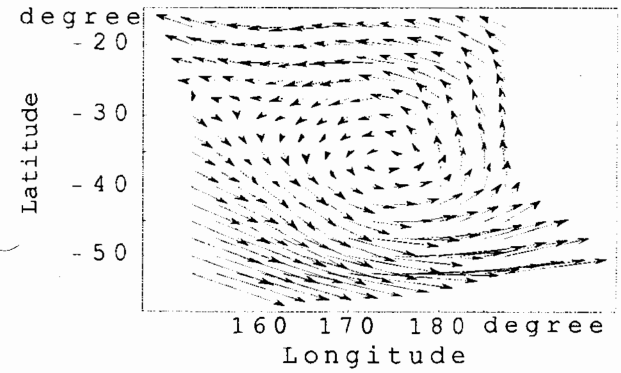
From the result in Fig. 8, it is obvious that our new methodology is quite useful strategy in order to reduce the noise vectors containing in the raw vortex flow with velocity on fluid dynamics.

3. Application to the Meteorological Data

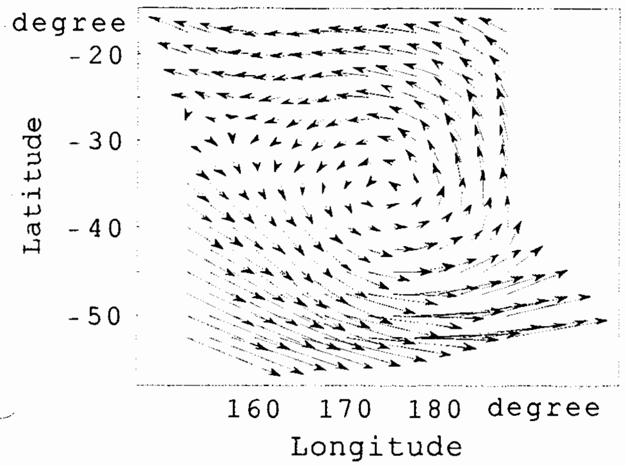
We have applied our methodology to the small scale meteorological data. Fig. 9 shows a practically measured wind vector distribution and its refined result.



(a) raw data



(b) local part of the raw data



(c) refined of (b)

Fig. 9 (a)Practically measured wind vector, (b)local part of the raw data(a) and (c) refined of (b).

4. CONCLUSION

As shown above, we have proposed a novel approach in order to remove the noise vector from vortex with velocity in fluid dynamics. At first, we have evaluated the vector and scalar potentials from two dimensional noisy raw vector flow based on the Helmhortz's theorem. Second, we have applied the discrete wavelet transform to each of vector and scalar potentials. Utilizing the data compression ability of the discrete wavelet transform, noise components included in the raw data have been potentials are reduced.

Simulation and practically measured vector applications have been demonstrated its usefulness.

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