Inverse Electromagnetic Problems by Field Visualization

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***Abstract*—The main inverse electromagnetic problems are formulated and solved for testing and inspecting of many devices such as thin-film transformers, inductors, printed circuit boards, motherboards of personal computers, etc. The visualization technique is applied to determine the current distributions and for nondestructive detection of the magnetic materials by locally measured magnetic fields. The image of the field distribution is processed using field theory. The color-source densities have been evaluated from the color distributions of the field data set. It was found that the color-source distribution corresponds to the field source distribution. Generating new color distribution with obtained color-source densities can essentially reduce the noise of the measured field data. With the proposed approach the quality of the images can be essentially improved. The results obtained show that various inverse electromagnetic problems can be successfully solved using visualized information of field distribution and image processing techniques.**

***Index Terms*—Image processing, inverse problems, nondestructive testing, visualization.**

I. Introduction

MODERN electronics and electromechanical devices require reliable methodologies for testing, control, and inspecting of their important parts. Determination of the current distribution of printed circuit boards (PCBs) and motherboards of personal computers, detection of magnetic materials, defects, and cracks in metallic parts without decomposition are reduced to inverse electromagnetic problems—identification, nondestructive testing (NDT), eddy-current testing (ECT), etc. Usually, magnetic field is measured and visualized over the parallel surfaces. Thus, the current distribution, magnetic materials, defect, and crack are determined from the locally measured magnetic field. Estimation of the field sources from field measurements outside the source area formulates the inverse source problem. As the field data are less than the number of unknown source data, the inverse problem is always ill-posed. Many different techniques are proposed to solve the inverse electromagnetic problems [1]–[5]. The field visualization facilitates modeling and analysis of the electromagnetic phenomena and processes. Recently, the field theory was applied effectively over the images for solving of various application problems [6]–[13].

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In this paper, the field theory and image processing are applied over the image, visualizing the field distribution. Several inverse electromagnetic problems for visualization of current distribution, shapes and locations of the magnetic materials, defects and cracks, etc., can be considered using visualized field distributions. The image of the field distributions is processed. The color-source distribution is obtained using the image color model. The color-source distribution corresponds to the field source distribution. Thus, visualizing the color-source distribution it is possible to visualize the current distribution or to determine the magnetic material shapes and locations.

The measured field database is usually insufficient or the data are accompanied with noise. Using the image color model and solving the inverse problem over the image it is possible to improve the quality and to change the characteristics of the image [6]. The generalized vector sampled pattern matching (GVSPM) method is applied to solve an ill-posed linear system of equations. The new color distributions are generated using the obtained color-source densities.

II. Formulation of Inverse Electromagnetic Problems

In inverse source problems, one seeks to estimate the source of a field that is observed outside the source area.

A. Current Distribution Determination

Most of the modern electronic devices such as thin-film transformers and inductors, PCBs, etc., have thin and compact shape. Thus, the current visualization is reduced to two-dimensional (2-D) current searching on a flat surface from the local magnetic field measurements at parallel surfaces to the current distributing surface. The color-source densities distribution obtained from the field distributions help in visualizing the current distribution.

B. Magnetic Material Detection

For nondestructive detection of magnetic materials in inaccessible region from the information on the surfaces the magnetic material may be regarded as simple layer distributions of imaginary currents on its boundary [13]. Thus, the determination of shapes and locations of magnetic materials can be reduced to inverse source searching for the imaginary current distributions. For visualization of the imaginary current distributions the exciting coil is used and magnetic field on the surface from simultaneous action of the coil currents and magnetic material is measured. The magnetic field caused by magnetic material is extracted and visualized. The color-source densities distribution is obtained using the image color model of the field distribution. Visualization of the color-source distribution corresponds to the shape and location of the magnetic materials.

C. Image Reconstruction and Enhancement

In order to reduce the noise accompanying the measured field data, an inverse color problem is formulated and solved over the images of the magnetic field distribution. The obtained color-source densities are used to generate new color distributions. It was found that the quality of the images is essentially enhanced. Thus, the image reconstruction and enhancement is reduced to the solving of the inverse problem over the image using field theory.

III. Image Color Model of the Field Distribution

The image is considered as 2-D distribution of color components—red, green, and blue (RGB). The 2-D Poisson equation is imposed to be satisfied by each of RGB color components with homogeneous open boundary conditions

$$\nabla^2 \mathbf{A} = -\sigma \tag{1}$$

where $\mathbf{A}$ and σ are any of the color components RGB and color-source densities, respectively. The color component A can be represented by Green's function G over image pixels

$$A = \frac{1}{4\pi} \int_S \sigma G dS. \tag{2}$$

In order to determine the color component source densities, we compose the system of equations

$$\mathbf{CX} = \mathbf{Y} \tag{3}$$

where $\mathbf{C}, \mathbf{Y}$, and $\mathbf{X}$ are the n by m system matrix, nth order column vector of the color component, and mth order column vector of unknown color-source densities, respectively. The $n \times m$ system matrix $\mathbf{C}$ we call "system matrix of relations."

For image processing, the color components as well as color-source densities have to be determined in the x-, y-pixels of area of interest. The RGB color components are separated. For each of them the processed area is extracted and systems of (3) are composed. Caused by the extracted area, the number of unknowns in (3) m is much larger then number of equations n. The systems of equations are linear and ill-posed.

The inverse problem for color-source determination is formulated. In order to solve the system of (3), the GVSPM method is applied [7], [8]. The color-source densities are determined. The new visualization of the color distributions is carried out from the evaluated color-source densities. Because of the solution error of the system of equations, it is possible the results not to be good. The image reconstruction and enhancement are estimated using correlation factor. The quality of the image obtained can be improved organizing iterative procedure. For the RGB color distributions, we recalculate the color-source distributions.

The linear system of (3) is composed, where the number of equations is equal to the number of unknowns. Solving (3), new values of the color sources are calculated. The new color distribution is determined and visualized utilizing obtained color sources. The iterative procedure continues until correlation between two consequent images becomes less than predefined limit. The image can be improved or repaired by extracting the processed part, determining the color-source distributions, and generating the new color distributions.

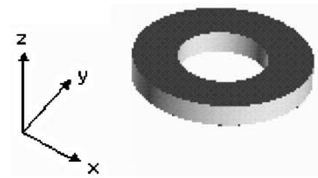


Fig. 1. Single circle coil.

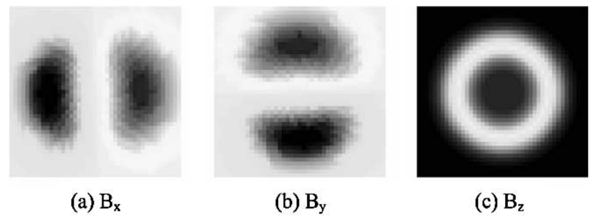


Fig. 2. Magnetic field density distributions (measured data of x, y, and z components).

During visualization of fields when the potential field distribution is considered, the RGB color distribution of the image corresponds to the potential distribution of the field. Applying the relation between vector potential $\mathbf{A}$ and flux density $\mathbf{B}$ over the pixels of the image

$$\mathbf{B} = \nabla \times \mathbf{A} \tag{4}$$

we obtain the RGB color flux-density distribution, which corresponds to the flux-density distribution of the field under consideration.

Also, the components of the magnetic flux densities are determined utilizing the color-source densities σ

$$\mathbf{B}_x = \mathbf{D}_x \sigma \tag{5}$$

$$\mathbf{B}_y = \mathbf{D}_y \sigma \tag{6}$$

$$\mathbf{B}_z = \mathbf{D}_z \sigma \tag{7}$$

where $\mathbf{D}_x, \mathbf{D}_y,$ and $\mathbf{D}_z$ represent the geometrical relations between pixels for corresponding components of the flux density. Thus, from the visualized potential distribution, it is possible to visualize flux-density distributions.

IV. Application

A. Current Distribution Determination

The magnetic field is measured over the parallel surface at distance $d = 0.005$ m from the single circle coil shown in Fig. 1. The inner and outer diameters are $d_1 = 0.02$ m and $d_2 = 0.025$ m, respectively. The coil height is $h = 0.005$ m. The components of the magnetic field densities $\mathbf{B}_x, \mathbf{B}_y$, and $\mathbf{B}_z$ are visualized and shown in Fig. 2. As the system is axisymmetrical, the radial and axial components of the magnetic field are determined and presented in Fig. 3. The RGB components for each component of magnetic flux densities are separated and presented in Figs. 4–6. The ill-posed inverse image problem is

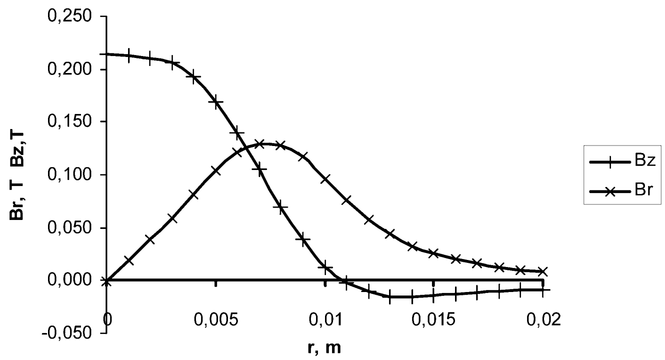


Fig. 3. Magnetic field density distributions—radial and axial components $\mathbf{B}_r, \mathbf{B}_z$.

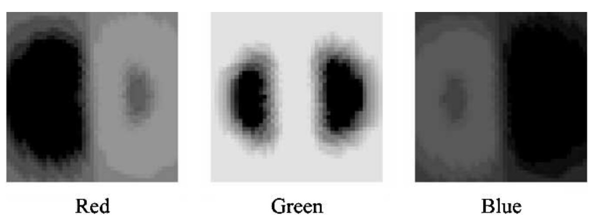


Fig. 4. Color component distributions of $\mathbf{B}_x$.

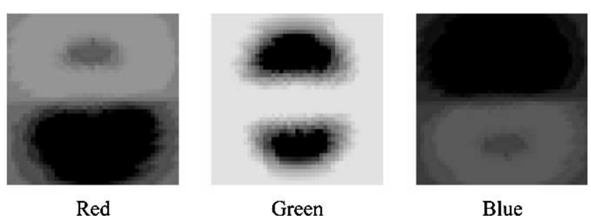


Fig. 5. Color component distributions of $\mathbf{B}_y$.

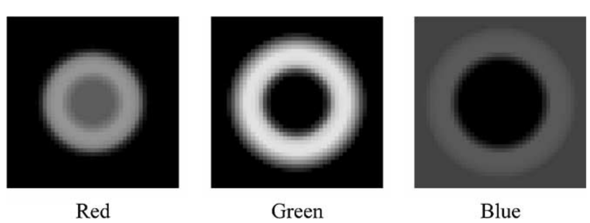


Fig. 6. Color component distributions of $\mathbf{B}_z$.

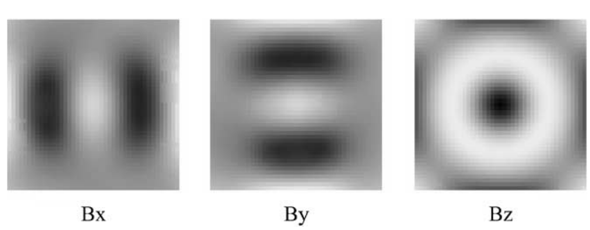


Fig. 7. Color-source density distributions.

formulated for determination of the color-source density distributions. The system of (3) for each color component—RGB is solved by the GVSPM method. The color-source density distributions are obtained and visualized in Fig. 7 using measured field data at distance $d = 0.005$ m from the surface. It was found that the color-source distribution well corresponds to the searched current distribution shown in Fig. 8. Fig. 9 shows the current distribution obtained using field data measured at distance $d = 0.005$ m, $d = 0.004$ m, and $d = 0.003$ m.

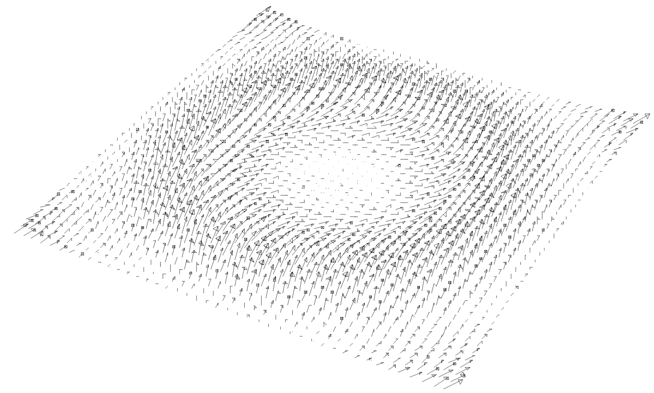


Fig. 8. Current distribution using data at distance $d = 0.005$ m over the coil.

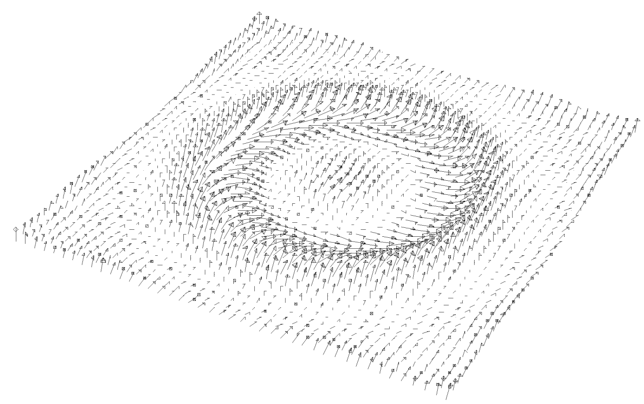


Fig. 9. Current distribution using data at distance $d = 0.005$ m, $d = 0.004$ m, and $d = 0.003$ m over the coil.

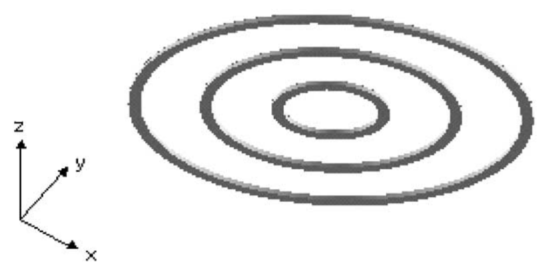


Fig. 10. Three thin turns coil.

The electromagnetic system composed of three turns with different voltage supply is shown in Fig. 10.

The diameters of the thin coil turns are $d_1 = 0.02$ m, $d_2 = 0.05$ m, and $d_3 = 0.08$ m. The magnetic field is measured over the parallel surface at distance $d = 0.002$ m. The components $\mathrm{B}_x, \mathrm{B}_y$, and B_z of the magnetic flux densities are visualized in Fig. 11. The radial and axial components of the magnetic field in cylindrical coordinate system are determined and presented in Fig. 12.

Using the image color model of the field distribution the color-source density distributions are obtained for each visualized component of the magnetic flux density at a distance of 0.002 m and visualized in Fig. 13. The corresponding current distribution is visualized in Fig. 14.

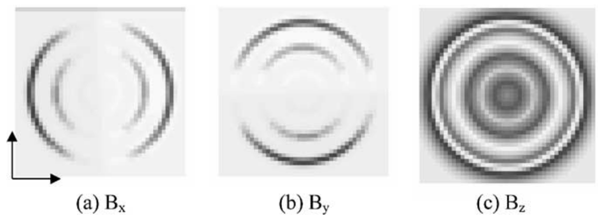


Fig. 11. Magnetic field density distributions.

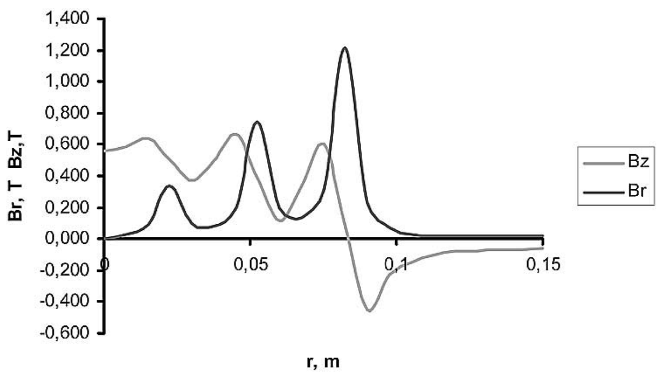


Fig. 12. Magnetic field density distributions—radial and axial components B_r, B_z.

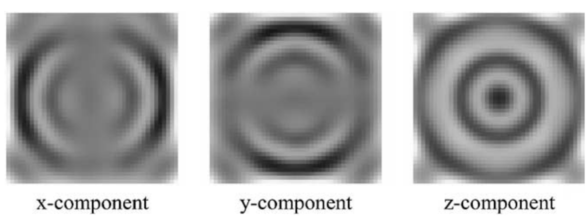


Fig. 13. Color-source distribution of the three thin turn coil.

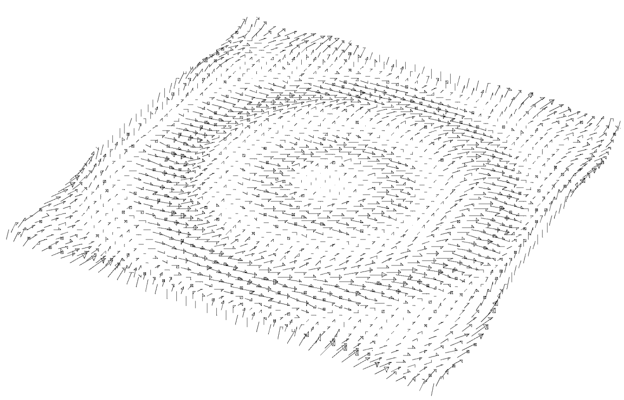


Fig. 14. Current distribution using data at distance $d = 0.002$ m over the three turns coil.

B. Nondestructive Detection of Magnetic Materials

For nondestructive detection of magnetic materials in inaccessible region by field measurements outside of the region the magnetic materials with different shapes are considered.

Fig. 15 shows the magnetic material with a rectangular shape. The exciting coil is used and magnetic field over the parallel surface at distance $d = 0.005$ m is measured and visualized in Fig. 16(a).

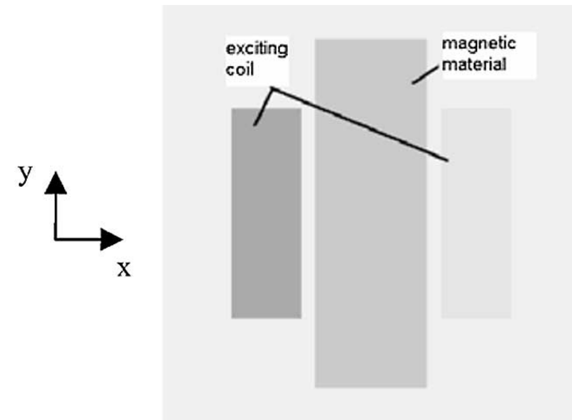


Fig. 15. Rectangular shape magnetic material.

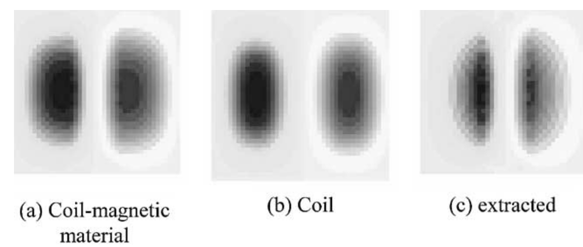


Fig. 16. Magnetic field distributions.

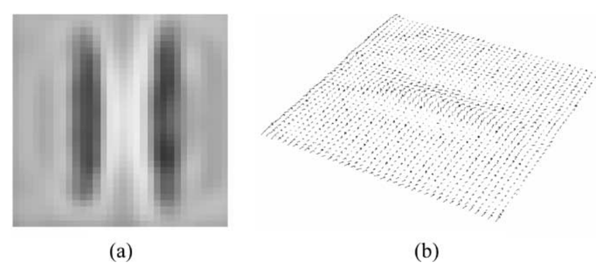


Fig. 17. Color-source distribution determined by magnetic material field data.

The magnetic field of the coil without magnetic material is determined and visualized in Fig. 16(b). The magnetic field caused from the magnetic material is extracted and shown in Fig. 16(c).

The image color model is applied over the image of the magnetic field distribution caused only by magnetic material.

The inverse image problem is formulated and color-source density distribution is obtained and visualized in Fig. 17. The part of the image with high intensities determines the high values of the color field sources which correspond to the simple layer of imaginary current distribution. The location of the simple layer of imaginary currents reveals the shape of magnetic materials.

The magnetic material with arbitrary shape is shown in Fig. 18. The resultant magnetic field produced using the exciting coil is visualized in Fig. 19(a). The magnetic field of the exciting coil is visualized in Fig. 19(b). The magnetic field caused only by magnetic material is extracted and visualized in Fig. 19(c).

The color-source density distribution for image of Fig. 19(c) is obtained using developed image color model and presented in Fig. 20. Analyzing the results obtained it was found that the

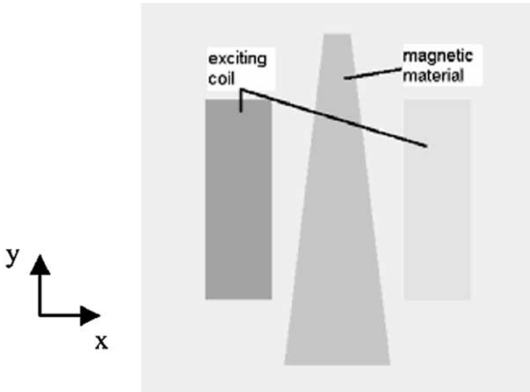
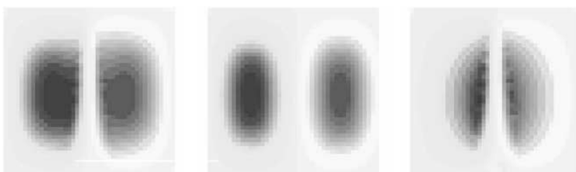
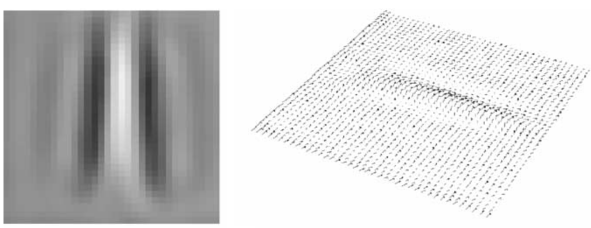


Fig. 18. Magnetic material with arbitrary shape.



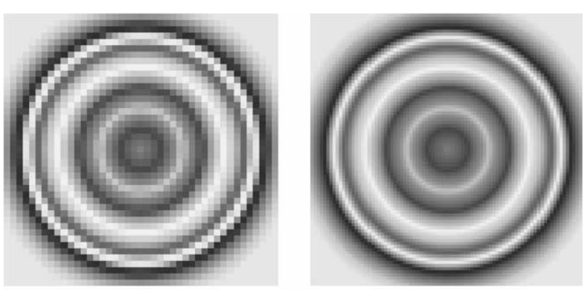
(a) Coil-magnetic material (b) Coil (c) extracted

Fig. 19. Magnetic field distributions.



(a) (b)

Fig. 20. Color-source distribution determined by magnetic material field data.



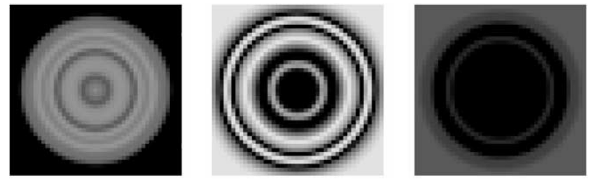
(a) original – B_z (b) improved – B_z

Fig. 21. Original and improved image of magnetic field density distribution (z-component).

part of the image with high color intensity determines the shape of magnetic material.

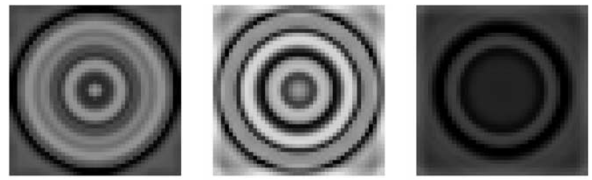
C. Image Reconstruction and Enhancement

In many cases, the available field data set is insufficient and is accompanied with noise. It is very difficult to analyze the visualized information properly using such data.



Red Green Blue

Fig. 22. Color component distribution of $\mathrm{B_z}$.



Red Green Blue

Fig. 23. Color-source density distribution of $\mathrm{B_z}$.

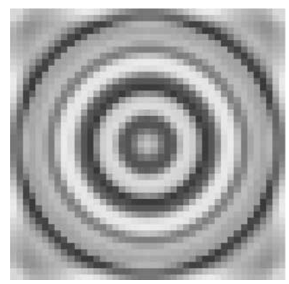


Fig. 24. RGB color-source density distribution.

In order to reduce the noise of the measured data set and to enhance the quality of the image, we formulate the inverse image problem for the color-source density distribution.

Applying the developed image color model, we separated the color components RGB for the image shown in Fig. 21(a) representing the z component of the magnetic flux-density distribution of the three thin turns coil in Fig. 10. The RGB color components are shown in Fig. 22.

The image in Fig. 21(a) is with low resolution. In order to increase the resolution or to remove the noise, we have to determine the color-source density distribution at high resolution.

We compose the ill-posed system of (3), where the number of equations is much less than the number of unknowns. With obtained color-source density distribution shown in Figs. 23 and 24, we generate new color distribution with better quality in Fig. 21(b).

The inverse approach described above and applied for processing of the magnetic field distribution images can be interpreted as an interpolation method of the available image data. The color distributions represented by (2) can be rewritten in matrix form as

$$\mathbf{A} = \mathbf{G}\sigma \tag{8}$$

where $\mathbf{G}$ is the system matrix of relations. The elements g of the system matrix $\mathbf{G}$ are calculated utilizing Green's function

$$g = \frac{1}{4\pi\mathbf{r}}. \tag{9}$$

It was found the correspondence exists between physical process that is visualized and the method used for image processing.

V. Conclusion

In this paper, we have applied the field visualization techniques based on the field theory for solving the main inverse problems in electromagnetics.

The image color model is developed. The Poison equation with open boundary condition is imposed. The color-source density distributions are calculated solving linear systems of equations by GVSPM. A pixel-oriented strategy has been utilized, which makes algorithms very fast. The new RGB color distributions are generated. The approach has a great potential and was successfully applied for image quality enhancement during magnetic field visualization.

The developed color image model was successfully applied for determination of current distribution from the field data locally measured at parallel surfaces. It was found that the color-source density distributions correspond to the current distributions.

The image processing technique based on the field theory was successfully applied to nondestructive detection of the magnetic materials. From the locally measured magnetic field distribution, it is possible to obtain and visualize the shape and location of the magnetic material. The magnetic field caused by the magnetic material is extracted and visualized. Visualization of the simple layer distribution of imaginary currents determines the shape and location of the magnetic material.

Solving the inverse color problem can essentially reduce the noise of the measured field data and enhance the quality of the images.

The results obtained reveal that the proposed image color model based on the field theory can be effectively applied for image processing in data set visualization of many other applications.

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