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# Image reconstruction for electromagnetic field visualization by an inverse problem solution

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**Abstract.** The purpose of this article is to apply an inverse approach for image reconstruction during electromagnetic field visualization. The reconstruction of the images is formulated as an inverse problem. For the image color model, the Poisson equation with open boundary condition is imposed. The color source densities have been evaluated from the color distributions for electromagnetic field data set. The Generalized Vector Sampled Pattern Matching method is applied to solve an ill posed linear system of equations for the corresponding inverse problem. The new color distributions are generated and visualized using the obtained color source densities. With the proposed inverse approach the dark parts of the images can be reduced or completely removed. This article collects several examples for electromagnetic field visualization of the most commonly used sensor coil.

## 1. Introduction

Visualization techniques facilitate the examination of unknown data sets and play an integral role in modeling and investigation of electromagnetic devices. The field distribution is of main importance considering many identification, NDT or EMC problems [1]. The Finite Element Method (FEM) or Boundary Element Method (BEM) are usually used for investigation of varies electromechanical devices. The results obtained are visualized using different visualization techniques. Sometimes the images are with not good quality or the data obtained do not correspond to the area of interest. To obtain new results, new FEM or BEM calculations are required which are accompanied by a lot of efforts and computational time. This is one of the main difficulties building interactive simulation and visualization systems. Many problems in medical diagnosis, as well as in industry quality control, require reducing the dark spots of the images. In order to reconstruct the images we propose an approach using formulation and solution of the inverse problem over the image. The image quality depends on the image resolution and color distribution. The image is considered as a 2D-distribution of color components – Red, Green, and Blue (RGB). The 2D-Poisson equation in Cartesian coordinate system is imposed to be satisfied by each of RGB color components with homogeneous open boundary conditions. In order to determine the color component distribution in the image part of interest we calculate the distribution of color component source densities utilizing Green functions. The inverse problem for color source densities

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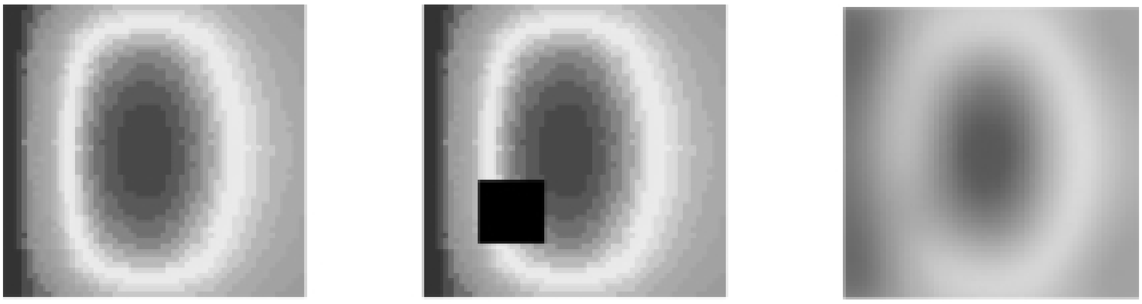


(a) Original image (b) Damaged image (c) Reconstructed image

Fig. 1. Original, damaged and reconstructed images of magnetic field distribution.

determination is formulated. The Generalized Vector Sampled Pattern Matching (GVSPM) method is applied to solve the ill posed linear system of equations [2]. New visualization of the color distribution is carried out from the evaluated color source densities. Several visualization examples demonstrate usability and applicability of the proposed inverse approach for visualization of electromagnetic field distributions as well as for image processing. The magnetic field distributions of the most commonly used sensor coil are considered.

## 2. Inverse problem for color source distribution

Let us consider the image color model building on the base of RGB colors. The image colors are considered as a 2D-distribution of RGB color components. A pixel-oriented strategy has been utilized. The 2D-Poisson equation in Cartesian coordinate system is imposed to be satisfied by each of RGB color components with homogeneous open boundary conditions

$$\frac{\partial^2 A}{\partial P_x^2} + \frac{\partial^2 A}{\partial P_y^2} = -\sigma, \tag{1}$$

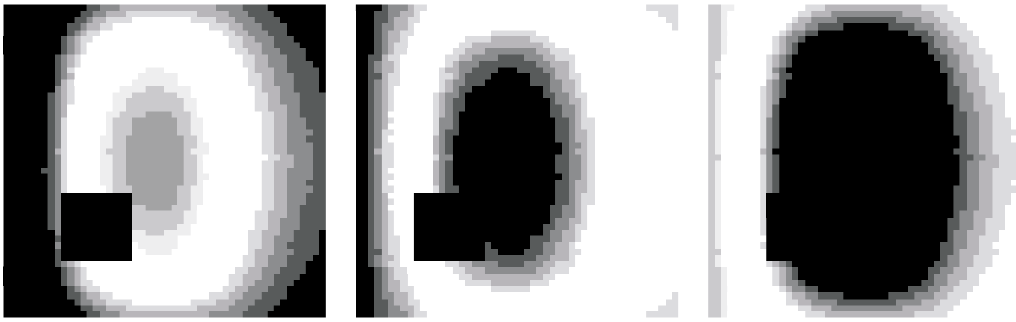
where $A$, $\sigma$, $P_x$ and $P_x$ are the any of the color components RGB, color source densities, pixels in $x$- and $y$-directions, respectively. The color component A can be represented utilizing the Green function over image pixels

$$A = \frac{1}{2\pi} \int_s \sigma \ln \frac{1}{r} dS, \tag{2}$$

where $r$ is the radius vector between the pixel of the color component $A$ and the pixel of the current integration, $S$ is the area of the image. In order to determine the color component source densities we compose the system of equations

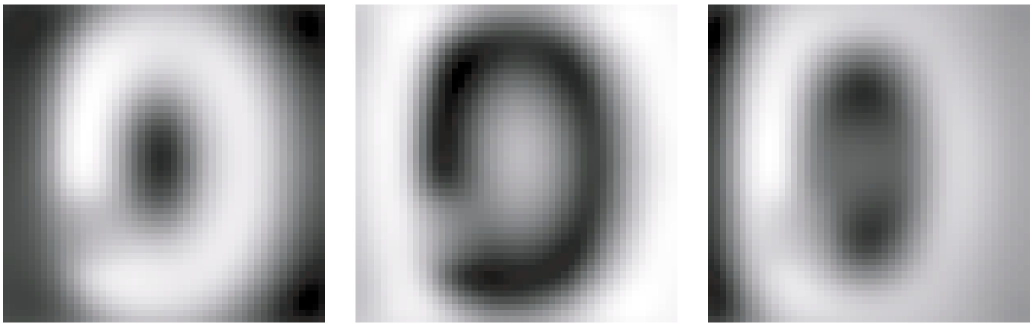
$$\mathbf{CX} = \mathbf{Y}, \tag{3}$$

where $\mathbf{C}, \mathbf{Y}$ and $\mathbf{X}$ are the $n$ by $m$ system matrix, $n$-th order column vector of the color component and $m$-th order column vector of unknown color source densities, respectively. To reconstruct the image, the color components as well as color source densities have to be determine in the x-, y-pixels of the area of interest. The RGB color components are separated. For each of them the damaged area is extracted and



(a) Red distribution. (b) Green distribution. (c) Blue distribution.

Fig. 2. Color distributions.



(a) Red source densities. (b) Green source densities. (c) Blue source densities.

Fig. 3. Color source distributions.

ill posed linear systems of Eq. (3) are composed. Caused by the extracted damaged area, the number of unknowns in Eq. (3) $m$ is much lager then number of equations $n$. The inverse problem for color source densities determination is formulated. In order to solve the systems of Eq. (3) the GVSPM is applied [2]. The new visualization of the color distributions is carried out from the evaluated color source densities. The image reconstruction is estimated using correlation factor. The quality of the image obtained can be improved organizing iterative procedure. For the RGB color distributions we recalculate the color source densities distributions. The well-posed linear systems of Eq. (3) are solved and new values of the color source densities are determined. The new color distribution is visualized utilizing obtained color source densities. The iterative procedure continues until correlation between two consequent images becomes less than predefined limit. The image can be improved or repaired by extracting the poor part, determining the color source densities distributions and calculating the new color distributions.

## 3. Practical visualization examples

The inverse problem formulated above has been applied for image processing during visualization of field distribution of electromagnetic devices. The magnetic field distribution of sensor coil has been

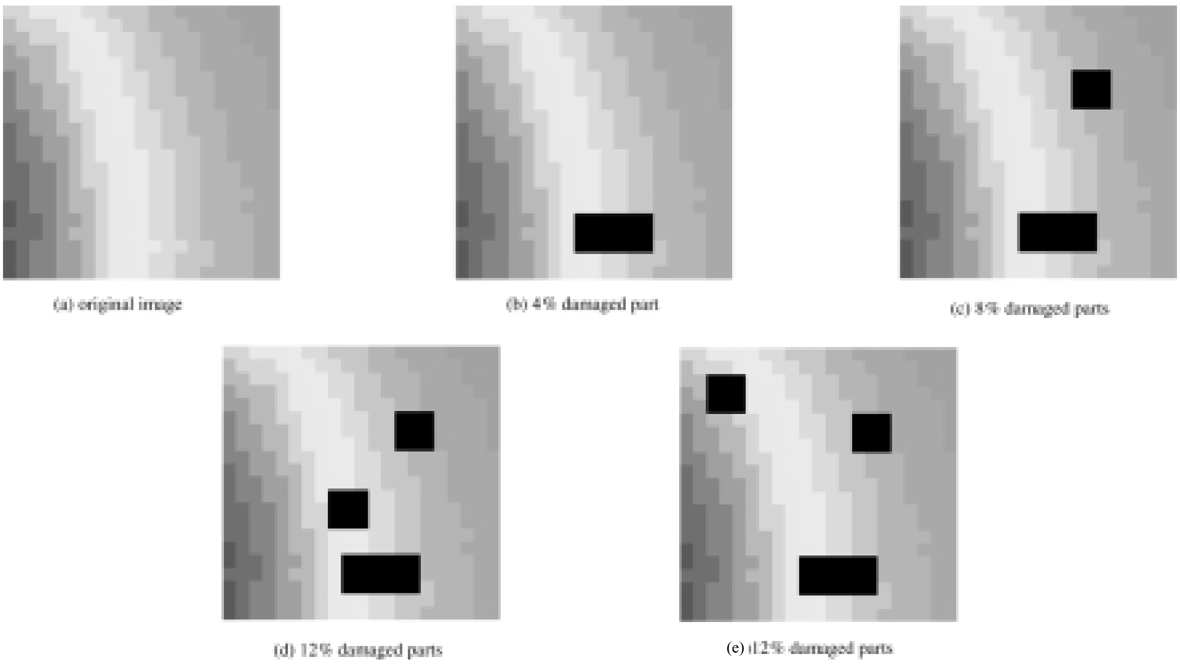



Fig. 4. Original image and images with damaged parts.

considered. The coil is of circular shape. The inner diameter is $r_1 = 0.01$ m, the outer diameter $r_2 = 0.03$ m and height $h = 0.04$ m. The magnetic field under consideration is axisymmetrical and can be easily visualized using different visualization tools. When the available image is damaged or corrupted it is difficult to analyze it properly. In order to reconstruct or repair the image we apply the proposed inverse approach. In Fig. 1 the original (a) and damaged (b) images of magnetic field distribution of the investigated sensor coil are given. The damage part is 5% of the entire image. The images are of low resolution. The RGB color distributions of the damaged image in Fig. 1(b) are shown in Fig. 2. The systems of equations Eq. (3) were solved for the three color components of the damaged image. The solutions of the systems represent the RGB color source densities over the image including damage region, shown in Fig. 3. The new visualization of the color distribution is carried out from the evaluated color source densities and the reconstructed image is obtained. The result is presented in Fig. 1(c). The correlation between original and reconstructed images is 0.93.

In many cases the region of interest is a small part of the image and it can be extracted and processed with proposed inverse approach instead of entire image. The part of the image, representing magnetic field distribution, has been extracted and presented in Fig. 4(a).

In Fig. 4(b)–(e) the damaged images are given. The damaged parts are 4%, 8%, 12% and 12%, respectively. The sizes and distribution of the damaged parts over the images are different. In order to reconstruct the damaged parts, the RGB color source densities are determined. The reconstructed images are presented in Fig. 5. The correlation factors $k$ are calculated between original and reconstructed images. The results obtained reveal that the quality of the processing essentially depends on the sizes and distribution of the poor parts. It is found that a good coincidence between original and reconstructed images exists. Considering only part of the image, the number of equations is substantially reduced as well as computational time. In Fig. 5(d)–(e) essential dark parts exist, caused by the relatively large damaged parts of the original image. In order to reduce them, for the image, shown in Fig. 5(e), we

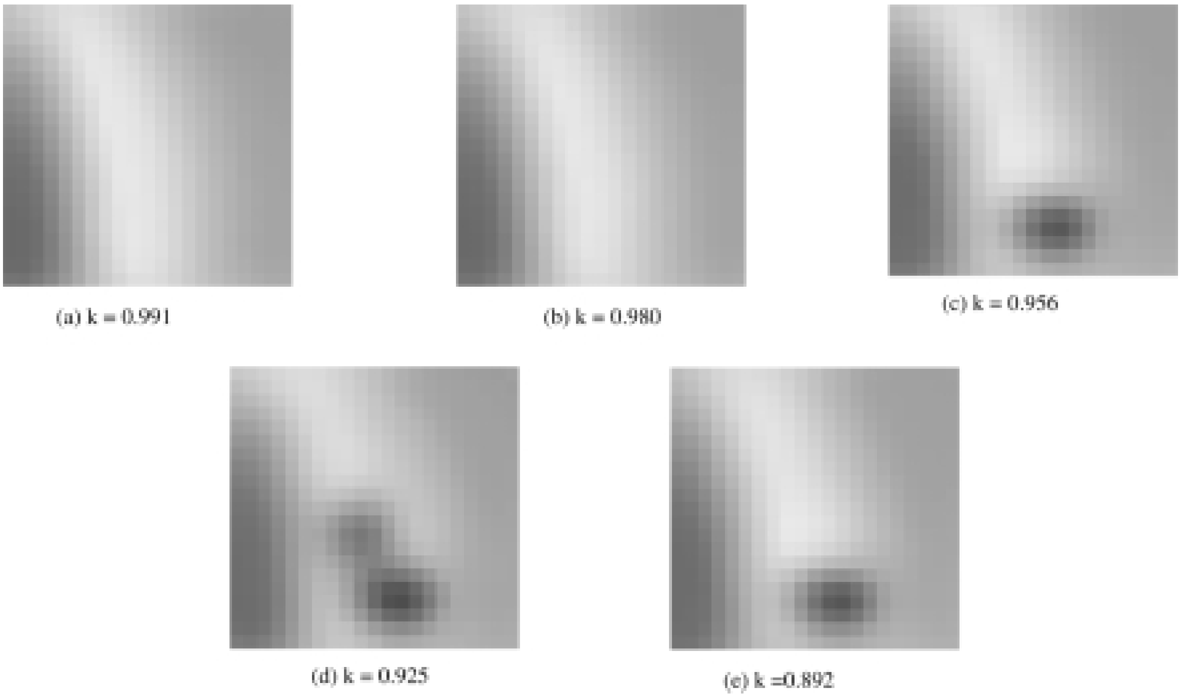
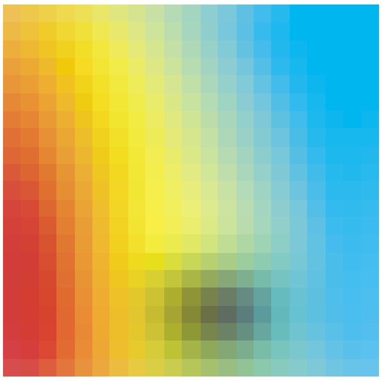



Fig. 5. Reconstructed images.

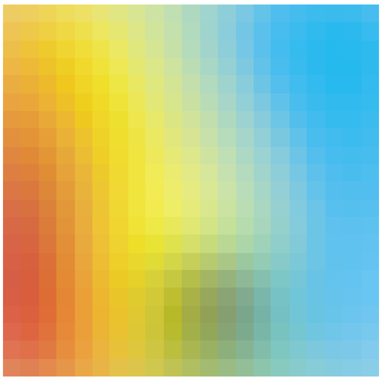
organize iterative procedure. The new RGB color distributions are generated, shown in Fig. 6(b). As it is presented in Fig. 6(b) and (c), the correlations between the images are very good. The dark parts of the images can be removed from reconstructed image by extracting the dark parts, as it is done for the RGB color components of the image in Fig. 6(c). The color source densities are determined in the dark part. The new color distribution is visualized and shown in Fig. 6(d). The correlation between Fig. 6(d) and Fig. 4(a) increases comparing with correlation of image of Fig. 6(c) and Fig. 4(a). The dark part is essentially reduced.

## 4. Conclusion

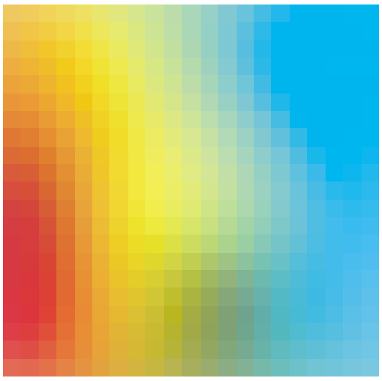
In this article, we have applied the inverse approach utilizing GVSPM for image reconstruction and improving the magnetic field distribution of sensor coil. For the image color model, the Poisson equation with open boundary condition is imposed. The color source densities distributions are calculated solving linear systems of equations by GVSPM. A pixel-oriented strategy has been utilized, which makes algorithms very fast. The new RGB color distributions are generated. The dark parts of the images can be reduced or completely removed. The low-resolution images can be improved by high-resolution color source determination. The inverse approach can be applied only over image parts of interest. That essentially reduces the computational expenses. The approach has a great potential and was successfully applied for quality improving and reconstruction of the images during magnetic field visualization. The results obtained reveal that the proposed inverse approach can be effectively applied for image processing in data set visualization of many other applications.



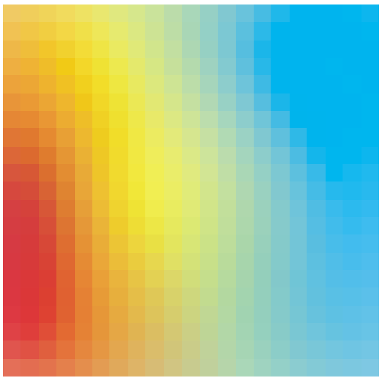
(a) k =0.892



(b) k = 0.956



(c) k=0.954



(d) k = 0.976

Fig. 6. Improved reconstructed images.

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