INVERSE PROBLEMS IN ENGINEERING MECHANICS
M. Tanaka, G.S. Dulikravich (Eds.)

OPTIMAL SENSOR LAYOUT FOR MINIMUM NORM APPROACH TO SEARCHING FOR RADIOACTIVE SOURCE DISTRIBUTION -ONE DIMENSIONAL CASE-

A.TAKEI[1], R. NISHIURA[2], T. OKA[2], S.HAYANO[1], and Y.SAITO[1]
1)College of Engineering, Hosei University, 3-7-2 Kajino Koganei, Tokyo 184, Japan
2) Industrial Electronics & Systems Laboratory MITSUBISHI ELECTRIC CORPORATION, 1-1, Thukaguchi-honmachi 8-chome,Amagasaki,Hyogo 661,Japan

ABSTRACT

Searching for the radioactive field source from the locally measured radioactive fields is essentially reduced into solving for an ill posed linear system.
In the present paper, we try to evaluate a unique solution of a simple one dimensional radioactive field source searching problem by means of the minimum norm method. Minimum norm method yields a unique solution whose norm takes a globally minimum value. However, the solution does not always correspond to a physically existing solution. To overcome this difficulty, this paper proposes an iterative approach finding out the optimal sensor layout.
As a result, we have succeeded in evaluating the unique radioactive field source distribution corresponding to the physically existing source.

KEYWORDS

Minimum norm method, inverse problem, optimum sensor layout

INTRODUCTION

With the developments of modern nuclear electric power plant, leakage of radioactive rays is one of the serious safeguard items. In order to protect and control the leakage of radioactive fields, all of the modern nuclear electric power plant installs various sensor and facilities.
Recently, a new radioactive sensor made of optical fiber is developed to improve the reliability of safeguard systems. This optical fiber type radioactive sensor can be installed to anywhere in the nuclear plant. Even if the radioactive fields are monitored, leakage radioactive field source should be identified to prevent the radioactive contamination. This means that the radioactive field source must be identified from the locally measured radioactive fields.
Searching for the radioactive field source from the locally measured radioactive fields is essentially reduced into solving for the inverse problems. In the other words, searching for the radioactive field source confronts to solving for an ill posed linear system. Various numerical method have been proposed solving for the ill posed linear systems. In biomagnetic field problems, the least squares and minimum norm methods are widely used[1-3]. The former is applied to finding the most dominant single field source, i.e. current dipole, and the latter is used to identifying the field source distributions. Further, new techniques called sample pattern matching has been proposed[4-6].
In the present paper, we try to evaluate a unique solution of a simple one dimensional radioactive field source searching problem by means of the minimum norm method. Minimum norm method yields a unique solution whose norm takes a globally minimum value. However, the solution depends on the nature of system matrix, i.e., the solution does not always correspond to a physically existing solution. To overcome this difficulty, it is essential to find out the optimal radioactive sensor layout. This paper proposes an iterative approach finding out the optimal sensor layout based on the correlative analysis between the computed and exact solutions.

As a result, we have succeeded in evaluating the unique radioactive field source distribution corresponding to the physically existing field source.

SYSTEM EQUATION

Generally, inverse problem is reduced into solving for a following system equations:

$$C\mathbf{X} = \mathbf{Y}, \tag{1}$$

where $\mathbf{Y}, \mathbf{X}, C$ are the measured field vector with order n, source vector with order m, and n by m rectangular system matrix, respectively. The elements of system matrix C depends on the Green's function of physical system.
The minimum norm solution of Eq.(1) is given by

$$\mathbf{X} = C^T[CC^T]^{-1}\mathbf{Y}. \tag{2}$$

RADIOACTIVE FIELD SOURCE SEARCHING

GREEN FUNCTION

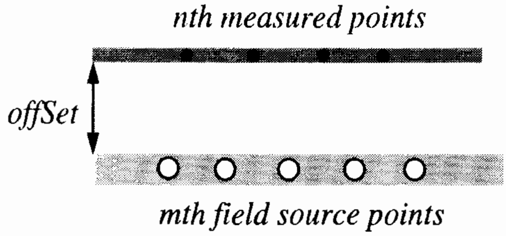
Let us consider a simple one dimensional inverse problem shown in Fig.1. The elements of the system matrix C in this case are determined by

$$G \propto \frac{1}{r_{ij}^2}, \tag{3}$$

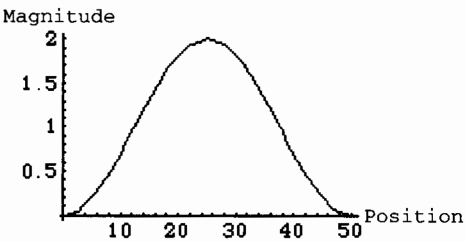
where r_{ij} is a distance between the measured field $i(=1,2,...,n)$and source point $j(=1,2,...,m)$.

MINIMUM NORM SOLUTION

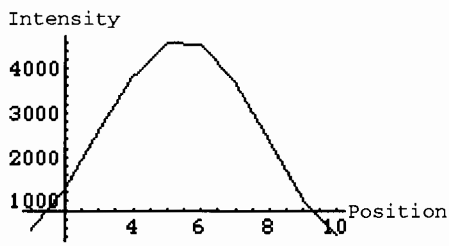
Figures 1(a) and 1(b) are the problem description and an exact model of the radioactive field source vector with order m=50, respectively. Figure 1(c) is a measured field vector with order n=10.Thereby, this problem is to solve a linear system having *10* equations and *50* unknowns. Simple application of the minimum norm method Eq.(2) to this problem yields a result shown in Fig.1(d), while an exact radioactive field source is shown in Fig.1(b).



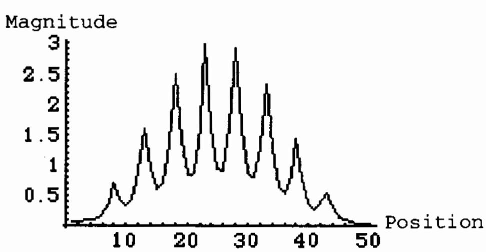
(a) Model description



(b)Model source distribution



(c) Measured field intensity

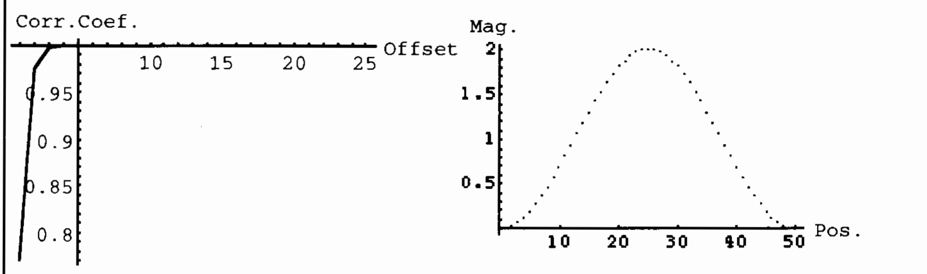


(d) Minimum norm solution

Fig.1. One dimensional radioactive field source searching problem and its minimum norm solution

CORRELATIVE ANALYSIS

After changing the offset between the field measuring and target surfaces, correlative analysis between the exact and estimated solutions was carried out. Figure 2(a) shows a convergence process. From the result in Fig. 2(a), it may be observed that the correlative coefficients over 5cm offset take a constant value. An offset (=9cm) taking the maximum correlative value yields a solution shown in Fig.2(b). It is obvious that the optimal sensor position could be found by the correlative analysis for the minimum norm strategy.



(a) Correlative coefficient vs. Offset (b) Evaluated field source at offset=9cm

Fig.2. Correlative analysis and the optimal solution

Further, uniqueness of the optimal offset is carefully examined. As a result, it has been confirmed that the maximum correlative coefficient value depends on the field source model, i.e. complexity of distribution, but the optimal offset takes a constant value. Figure 3 shows a relationship between the spatial frequency and correlative coefficient. From this result, it is obvious that the correlative coefficient takes smaller in value corresponding to the higher spatial frequency.

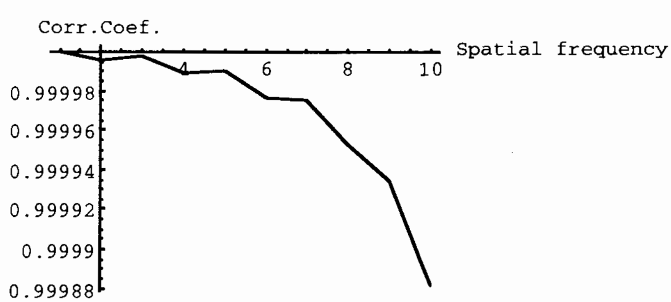
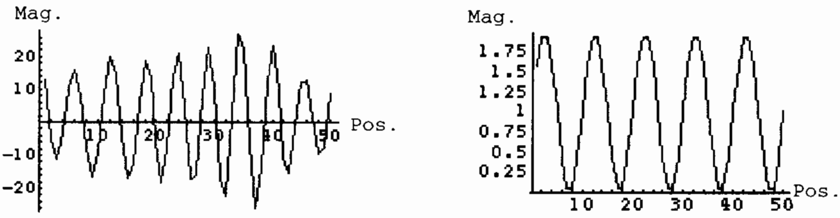


Fig. 3.Example of a relationship between the correlative coefficient and spatial frequency.

NOISE FIELDS

Practical measurement of the radioactive fields always accompanies the noise fields. This yields a serious problem to the minimum norm strategy. For example, only 1% random noise yields the fields source shown in Fig.4(a), while an exact model field source is shown in Fig. 4(b).



(a) Field source evaluated from 1% random noise (b) Model field source

Fig.4. Comparison the noise field source and exact one.

The noise field source obviously contains higher frequencies than those of exact field source. In order to reduce the noise fields, we applied the Fourier analysis. Figure 5 shows a Fourier spectrum of the field source shown in Fig.4(a).

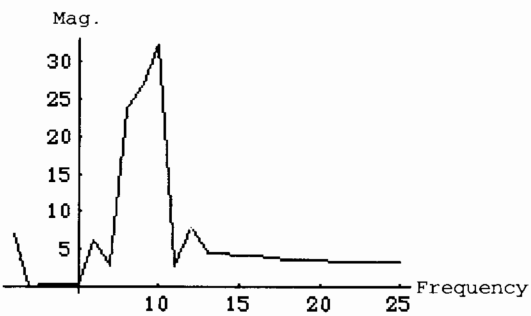


Fig. 5. Fourier spectrum of the field source shown in Fig.4(a).

Thus, removing over the 5th order frequency components from the result in Fig. 4(a), we can obtain an improved result. This is because the number of field measured points n=10 is able to include the first 5th order frequency components, i.e., 0,1,2,3 and 4th order frequencies. Figure 6 shows the improved solution together with exact one.

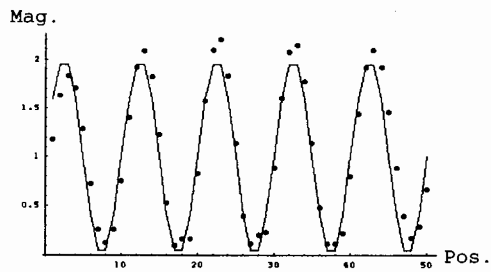


Fig. 6. Filtered solution by means of the Fourier analysis. Solid and dotted lines show the exact and computed solutions, respectively.

CONCLUSION

The minimum norm solution depends on the elements of coefficient matrix. In order to obtain the physically existing solution, the magnitude of matrix elements has been systematically changed by altering the sensor position. The correlative coefficients between the the computed and exact solutions have yielded the optimal sensor position. Further, we have proposed one of the ways to reduce the effect of noise fields by means of the Fourier approach.

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