INVERSE PROBLEMS IN ENGINEERING MECHANICS
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ESTIMATION OF CURRENT DISTRIBUTION WITHIN CONDUCTORS BY FIELD MEASUREMENTS

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ABSTRACT

In this paper, the current distribution in conductors is estimated from external magnetic field measured near the conductor surfaces. The inverse eddy current problem is formulated. The Sampled Pattern Matching (SPM) method has been used previously to solve inverse source and identification problems. Here, the same technique is extended to the inverse eddy current problems through the defining the governing equations for eddy current problem and using appropriate fundamental Green functions. The method is demonstrated by investigation of thin film transformer. The current distribution within thin film conductors from field measurements has been estimated at different mode and conditions of film transformer.

KEYWORDS

Inverse problem, current distribution, eddy current, thin film transformer, conductor.

INTRODUCTION

The current distribution in conductors is one of the most important property which determine the quality and performance of the electromagnetic devices (transformers, reactors, inductors, converters, AC transmission lines etc.) especially at high frequency. The current distribution in conductors is of great importance in design and optimization of such devices. Determining the current distribution in conductors in order to obtain prescribed external or internal magnetic field densities is of main interest considering nondestructive testing (NDT) or electromagnetic compatibility (EMC) problems.
In some electromagnetic devices the multiconductor systems are voltage supplied. In this case the total currents in the conductors as well as the current distribution are of unknown values. In other problems the total currents in the conductors are given. Then the source current densities are unknown. The problem becomes more complex when eddy currents are considered. These topics have recently become a subject of interest of several papers [1,8].

In this paper, data of external magnetic field measured near the conductor surfaces are used in order to estimate the current distribution in conductors. The inverse eddy current problem is formulated. Recently, the Sampled Pattern Matching (SPM) method was developed as a powerful tool to solving inverse source and identification problems [9-13]. Here, the same technique is extended to the inverse eddy current problems through defining the governing equations for eddy current problem and using appropriate fundamental Green functions. The current distribution problem is reduced to a position searching problem of the current sources in conducting regions from field distributions obtained by measurements. Thin film transformer is investigated. Previously, this transformer has been analyzed using integral equations [14]. Magnetic field distribution around the surfaces of the conductors is measured and used as an input data to SPM method. The cross sections of the conductors are considered. The conducting region is considered as composed of source and eddy current carrying regions. The current distribution within thin film conductors has been estimated at different mode and conditions of the film transformer.

FORMULATION OF THE INVERSE PROBLEM

Governing Equations for Eddy Current Problem.

The governing differential equation for steady-state eddy current problem are given by the following diffusion equation, assuming the gauge $\nabla \mathbf{A} = \mathbf{0}$

$$-\frac{1}{\mu}\nabla^2 \mathbf{A} = \mathbf{J}_s - \sigma\frac{\partial \mathbf{A}}{\partial t}, \tag{1}$$

where $\mathbf{A}$ is the magnetic vector potential, $\mathbf{J}_s = -\sigma\nabla V$ is the imposed source current density, μ is the magnetic permeability, σ is the electric conductivity, V is the scalar electric potential. Considering time-harmonic electromagnetic field and using phasor representation (1) is represented by

$$\nabla^2 \mathbf{A} + k^2 \mathbf{A} = -\mu \mathbf{J}_s, \tag{2}$$

where $\mathbf{A}$ is the complex magnetic vector potential, $k^2 = -j\omega\mu\sigma$ is the complex diffusion constant, ω is the angular frequency and $j = \sqrt{-1}$.
The total current density can be considered as composed of two components

$$\mathbf{J} = \mathbf{J}_s - j\omega\sigma\mu\mathbf{A}. \tag{3}$$

Let define the Green's function

$$G(\mathbf{r}) = \frac{e^{-jkr}}{(4\pi\,\mathbf{r})}, \tag{4}$$

as a fundamental solution which satisfies the following differential equation

$$\nabla^2 G(\mathbf{r}) + k^2 G(\mathbf{r}) = -\delta(\mathbf{r}), \tag{5}$$

where $\delta(\mathbf{r})$ is the Dirac Delta function and $\mathbf{r}$ is the radius-vector between the source point and the field point.
The solution to (2) in unbounded space is expressed as

$$\mathbf{A} = \frac{\mu_0}{4\pi}\int_V \mathbf{J}G(\mathbf{r})dV, \tag{6}$$

where μ_0 is the magnetic permeability of vacuum.
The magnetic flux density obtained from magnetic vector potential **A** is

$$\mathbf{B} = \nabla\times\mathbf{A} = \nabla\times\frac{\mu_0}{4\pi}\int_V \mathbf{J}G(\mathbf{r})dV. \tag{7}$$

In regions free of the eddy currents, the static magnetic field can be expressed as the *curl* of magnetic vector potential satisfying the differential equation

$$-\frac{1}{\mu_0}\nabla^2\mathbf{A} = \mathbf{J}_s. \tag{8}$$

If **C** is n by m matrix and **F** is n-th order column vector of the vector potential **A** or of the magnetic field density **B**, then (6) or (7) can be presented by a system of equations in matrix form

$$\mathbf{CJ} = \mathbf{F}. \tag{9}$$

Usually, the investigated conducting region is composed of regions with imposed currents and regions with induced currents. Then, let refer to the conducting regions with impose currents as source current regions as well as to the conducting regions free from imposed currents as eddy current regions. Furthermore, the magnetic vector potential **A** using (6) and magnetic field density **B** using (7) are caused by the action of the source and eddy currents. Then, the system of equations (9) is expressed as

$$\mathbf{C}_s\mathbf{J}_s + \mathbf{C}_e\mathbf{J}_e = \mathbf{F}, \tag{10}$$

where the matrix $\mathbf{C}_s$ and current density $\mathbf{J}_s$ correspond to the source current regions and $\mathbf{C}_e$ and current density $\mathbf{J}_e$ corresponds to the eddy current regions, respectively.
From system of equations (10) and using magnetic field distribution outside of the conducting regions, the estimation of current distribution can be reduced to a position searching problem of the current densities in the source and eddy current regions. As magnetic field distribution is obtained by local field measurements then the number of equations of (10) is much less than the number of unknowns of the current densities. The defined inverse source searching problem is ill-posed and it is difficult to obtain an unique solution.

Sampled Pattern Matching Method

Sampled Pattern Matching method is applied to estimate the current distribution in source and eddy current regions. The pattern matching figure based on the angle between two vectors is used

$$\gamma_j = \frac{\mathbf{F}^T.\mathbf{c}_j}{\|\mathbf{F}\|\|\mathbf{c}_j\|}. \tag{11}$$

The maximum value of γ shows that the direction of the vectors **F** and $\mathbf{c}_j$ coincides. This property of the pattern figure γ (11) is used as a criterion to estimate the pilot point position and to find the pilot point solution of the inverse source searching problem applying SPM method.

As induced currents flowing in eddy current regions are caused by the currents flowing in source current regions a couple pattern vectors $\mathbf{c}_j, \mathbf{c}_k$ from source and eddy current regions is searched where the pattern matching figure γ_{jk} (12) takes a maximum value of

$$\gamma_{jk} = \frac{\mathbf{F}^T \cdot (\mathbf{c}_j + \mathbf{c}_k)}{\|\mathbf{F}\| \|\mathbf{c}_j + \mathbf{c}_k\|}, \qquad \begin{array}{l} j = 1,2,\ldots,m_s; \\ k = 1,2,\ldots,m_e. \end{array} \tag{12}$$

where m_s and m_e are numbers of estimation points from source and eddy current regions, respectively.

If a point h from source current regions and a point t from eddy current regions take the maximum of γ_{jk} (12) then h and t are the first pilot points for the source and eddy current regions, respectively. The vectors $\mathbf{c}_h$ and $\mathbf{c}_t$ are the first pilot patterns for corresponding regions.

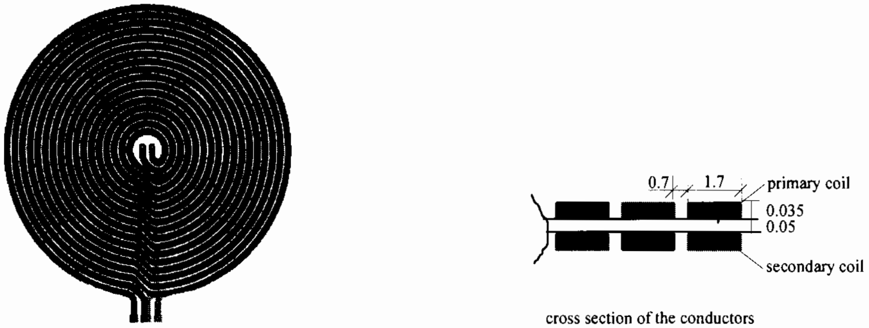
The process continues determining a point u from source current regions and a point v from eddy current regions where γ_{jk} takes a maximum of

$$\gamma_{jk} = \frac{\mathbf{F}^T \cdot (\mathbf{c}_h + \mathbf{c}_t + \mathbf{c}_j + \mathbf{c}_k)}{\|\mathbf{F}\| \|\mathbf{c}_h + \mathbf{c}_t + \mathbf{c}_j + \mathbf{c}_k\|} \qquad \begin{array}{l} j = 1,2,\ldots,m_s; j \neq h; \\ k = 1,2,\ldots,m_e; k \neq t. \end{array} \tag{13}$$

Then u and v are the second pilot points for the source and eddy current regions, respectively. A similar process to (12-13) is continued up to the first peak of γ . The pilot point solution is obtained. The high values of the γ correspond to the dominant positions of the field sources $\mathbf{F}$ distributed within source and eddy current regions.

APPLICATION

The current distribution within conductors of a thin film transformer has been estimated by using data of external magnetic field measured near the conductor surfaces. Thin film transformer under consideration is shown in Fig. 1a. The sizes of the conductors and the



(a) (b)

Fig. 1. Thin film transformer: (a) Thin film transformer; (b) Sizes of the conductors in mm.

distances between them are presented in Fig. 1b. The primary and secondary coils of the transformer are composed of two laminated layers of thin film conductors. The layers of the separate coils are connected in series. The transformer can be considered as a voltage supplied multiconductor system. The operation principle of the transformer is based on the skin and proximity effects at high frequency. In this case the current density distribution as well as total currents in the respective conductors are of unknown values. The current distribution is of main interest as the integral characteristics and parameters of the electromagnetic device can be easily and precisely evaluated.

The film transformer as well as its magnetic field are accepted with axial symmetry. The cross sections of the conductors in ρz -plane are considered. The axis of symmetry lies on the left of the conductors. The cross-section of the conductors is divided in rectangular elements. The constant current density is assumed within each element. The estimation points are located in the middle of the elements.

In order to estimate the current distribution in transformer conductors, considering steady-state skin effect problem, the magnetic field around the transformer is measured at surfaces located just above and bellow the conductor surfaces. The investigations are made at frequency 100kHz. Here, we use inversions of numerically simulated experimental data, rather then real ones.

Considering cross-section of the conductors the scale in z-direction is reduced in order to present the current source distribution within conductors. The black and gray colors are used to current distribution in source and eddy current regions, respectively.

The SPM method is applied and distribution of current sources is estimated within source and eddy current regions of the conductors.

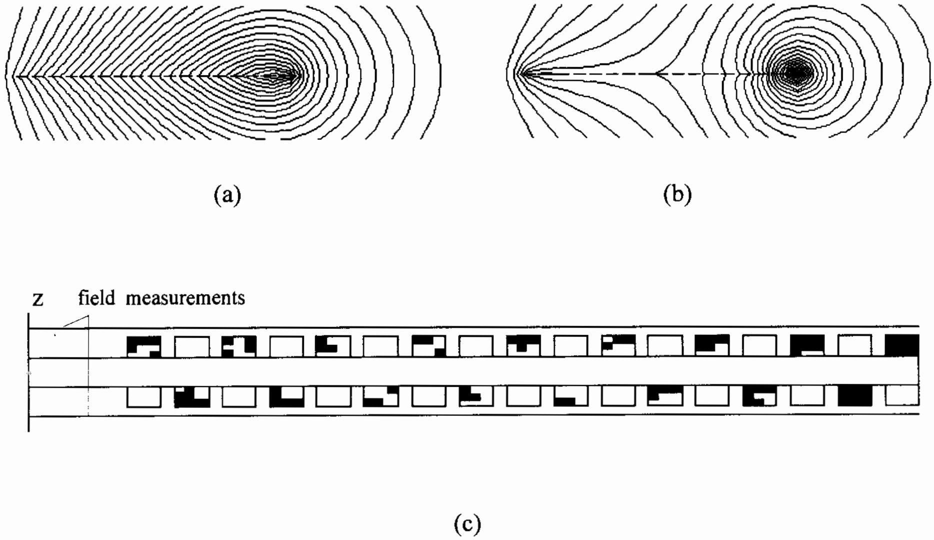
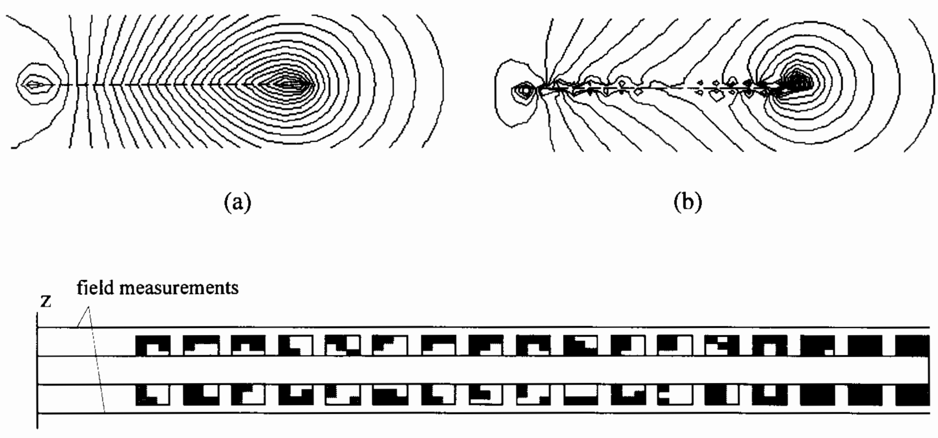


Fig. 2. Thin film transformer with open secondary circuit: (a) Magnetic flux lines (real part); (b) Magnetic flux lines (imaginary part); (c) Distribution of current sources in conductors.



(a) (b)

(c)

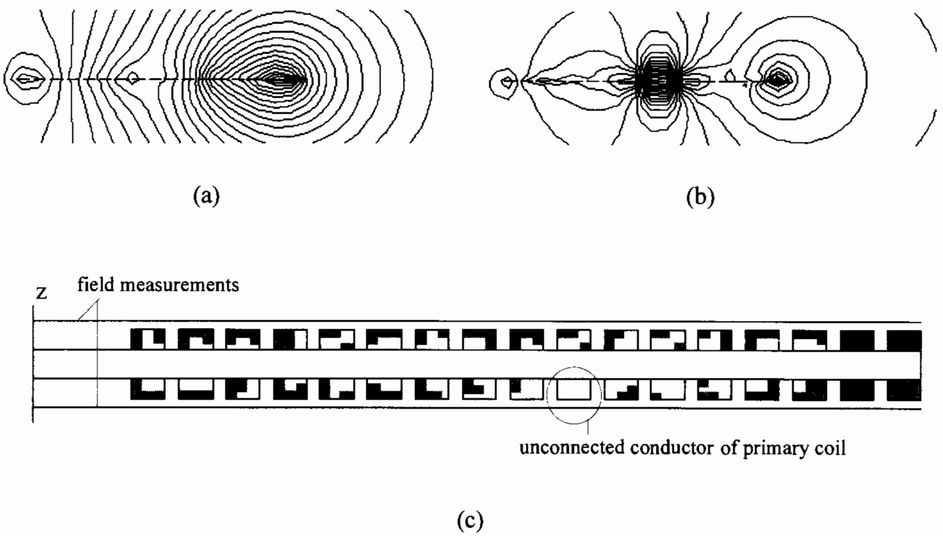
Fig. 3. Thin film transformer with resistive load of the secondary circuit: (a) Magnetic flux lines (real part); (b) Magnetic flux lines (imaginary part); (c) Distribution of current sources in conductors.

Thin film transformer with open secondary circuit is considered. The primary coil is voltage supplied and its conductors compose the source current regions. In this case no eddy current regions exist. Magnetic field around conductors is caused by the primary coil excitation. The magnetic flux lines are shown in Fig. 2a,b. Using data of magnetic flux density and applying the SPM method it is determined the conductors where the current sources are located.

The current distribution in conductors is presented in Fig. 2c. It is shown that the current sources are located only in the conductors belonging to the primary coil of the transformer. No current sources exist in secondary coil conductors where no current flows.

Thin film transformer with resistive load of the secondary circuit is considered. The primary coil conductors compose the source current regions and the secondary coil conductors compose the eddy current regions. Magnetic field around conductors is caused by the primary and secondary coil currents. The real and imaginary part of magnetic flux lines are shown in Fig.3a,b. The distribution of current sources in conductors is determined and presented in Fig. 3c. It is shown that the current sources are located in the conductors belonging to the primary and secondary coils of the transformer.

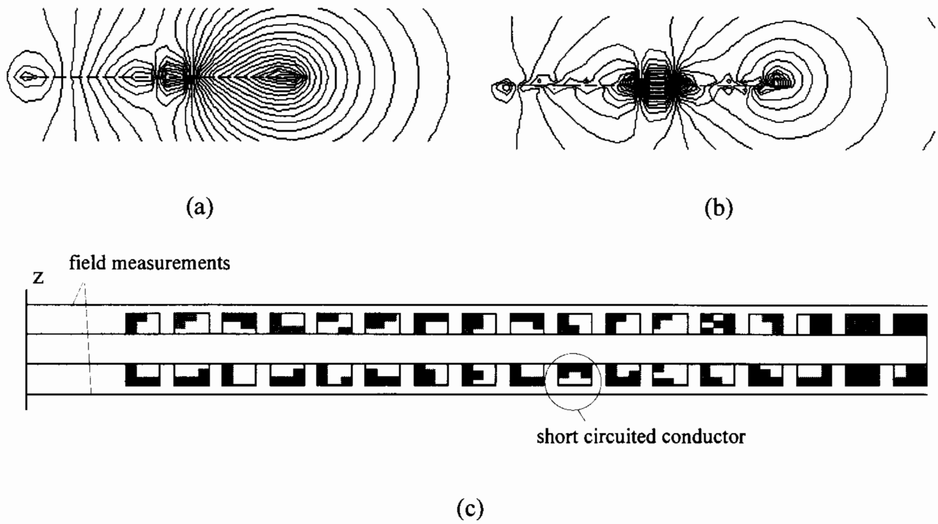
In practical use of multiconductor devices often one or more conductors are not connected or damaged. In this case the current distribution in conductors changes and cause change of external magnetic field around conductors. From data of field measurements it is possible to determine the current distribution in conductors and to find the conductors where no current flows. Let consider a film transformer where a conductor of the primary coil is not connected.



(a) (b)

(c)

Fig. 4. Thin film transformer with resistive load of the secondary circuit and an unconnected conductor of the primary coil: (a) Magnetic flux lines (real part); (b) Magnetic flux lines (imaginary part); (c) Distribution of current sources in conductors.



(a) (b)

(c)

Fig. 5. Thin film transformer with resistive load of the secondary circuit and a short circuited conductor: (a) Magnetic flux lines (real part); (b) Magnetic flux lines (imaginary part); (c) Distribution of current sources in conductors.

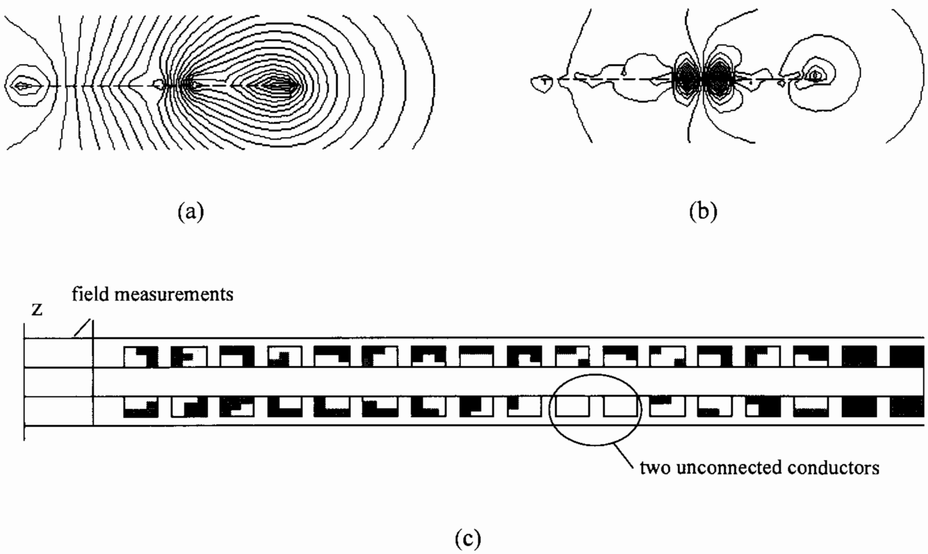


Fig. 6. Thin film transformer with resistive load of the secondary circuit and two unconnected conductors: (a) Magnetic flux lines (real part); (b) Magnetic flux lines (imaginary part); (c) Distribution of current sources in conductors.

Then the magnetic flux lines are presented in Fig. 4a,b. From field measurements the current source distribution in conductors is obtained and presented in Fig. 4c. It is shown that the current sources are situated in the conductors where the current is flowing. It is found that no current sources exist in unconnected conductor.

If there is a short-circuited turn then the magnetic field distribution is changed as shown in Fig. 5a,b. The current source distribution obtained is shown in Fig.5c. Eddy currents flow in short circuited conductor.

In Fig. 6 it is shown the magnetic flux lines and current distribution in case when two conductors are not connected. Using the magnetic field data and SPM method it is found that the current sources are located in source and eddy current regions and no current flows in the unconnected conductors.

Considering the inverse source problem of current distribution in conductors and applying SPM method it is quite important the quantity and quality of the field measurements around the conductor surfaces. It has been found that using the field measurements from different level of observation leads to good results.

CONCLUSION

In this paper, the estimation of current distribution in conductors is reduced to solving inverse source problem. The SPM method has been applied to determine the current source locations in conducting regions. The appropriate fundamental Green functions are introduced to the SPM method. The inverse eddy current problem is solved investigating the thin film transformer. The current distributions at open and load secondary circuit conditions of the transformer have been estimated using external magnetic field measured near the conductor surfaces. Determining the current distribution in conductors it is possible to find the unconnected conductors where no current flows as well as short circuited turns where eddy current flow. Considering the inverse source problem of current distribution in conductors and applying SPM method it is quite important the quantity and quality of the field measurements around the conductor surfaces. It has been found that using the field measurements from different level of observation leads to good results. It has been shown that the SPM can be used successfully to estimate the current distribution of multiconductor devices using magnetic field measurements.

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