INVERSE PROBLEMS IN ENGINEERING MECHANICS
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Wavelets Strategy for Ill Posed Linear Systems

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ABSTRACT

This paper proposes a new methodology solving for the ill posed linear systems. Key idea is that the system matrix is regarded as an image data. Applying the discrete wavelet transformation to this system matrix yields an approximate inverse matrix. Thus, we succeeded in solving the ill posed linear systems. An example concerning with the inverse problem in magnetostatic fields demonstrates the usefulness of our methodology.

KEYWORDS

Inverse problems, Discrete Wavelets Transform, Ill posed Linear System

INTRODUCTION

Most of the inverse problems appearing in engineering and science are reduced into solving the ill posed linear systems. Various numerical methods have been proposed for solving the ill posed systems. In biomagnetic fields, the least squares and minimum norm methods are widely used[1,2,3]. The former is applied to finding the most dominant single field source, i.e. current dipole, and the latter is used to identifying the field source distributions. Further, new techniques based on the neural networks have been proposed[4,5].

On the other side, discrete wavelets transform method has been proposed in order to carry out the wave form analysis as well as image data compression. Wavelets transform makes it possible to collect the dominant elements of the image data in a particular region of the image data spectrum. Previously, we have succeeded in obtaining the approximate solutions of ill posed linear systems by applying the wavelets transform[6,7]. In this solution strategy, the two dimensional wavelets transform is applied to the system matrix which is regarded as one of the two-dimensional image data, then collecting the most dominant elements on the system spectrum matrix yields an approximate inverse matrix in the wavelet spectrum domain. Inverse wavelet transform of this approximate inverse matrix gives an approximate inverse matrix of the ill posed system. Thus, we have succeeded in obtaining the approximate solution of ill posed linear systems.

In this paper, we apply the higher order analyzing wavelets to the ill posed linear systems. Several techniques are proposed to overcome the serious problems that are confronted to the higher order wavelets application to the ill posed linear systems. As a result, we have succeeded in obtaining the approximate solutions with higher accuracy.

BASIC WAVELETS SOLUTION

Let us consider a typical ill posed system of equations

$$\mathbf{Y} = C \cdot \mathbf{X}, \tag{1}$$

where $\mathbf{Y}, \mathbf{X}, C$ are the n-th order input, m-th order output vectors and n by m system matrix, respectively. The number of equation n is generally smaller than the number of unknowns m in the inverse problems so that Eq.(1) is a typical ill posed system of equations.

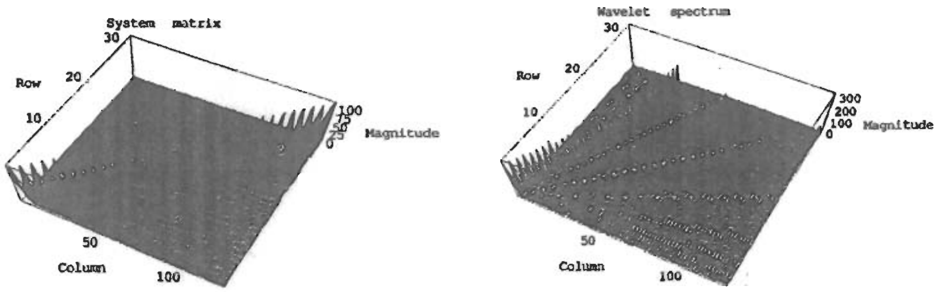
Wavelets transform to Eq.(1) is carried out by

$$\mathbf{Y}' = C' \cdot \mathbf{X}', \tag{2}$$

$$\mathbf{Y}' = W_n \cdot \mathbf{Y}, \qquad C' = W_n \cdot C \cdot W_m^T, \qquad \mathbf{X}' = W_m \cdot \mathbf{X}, \tag{3}$$

where W_n, W_m are the n and m-th order wavelets orthogonal transformation matrices, respectively.

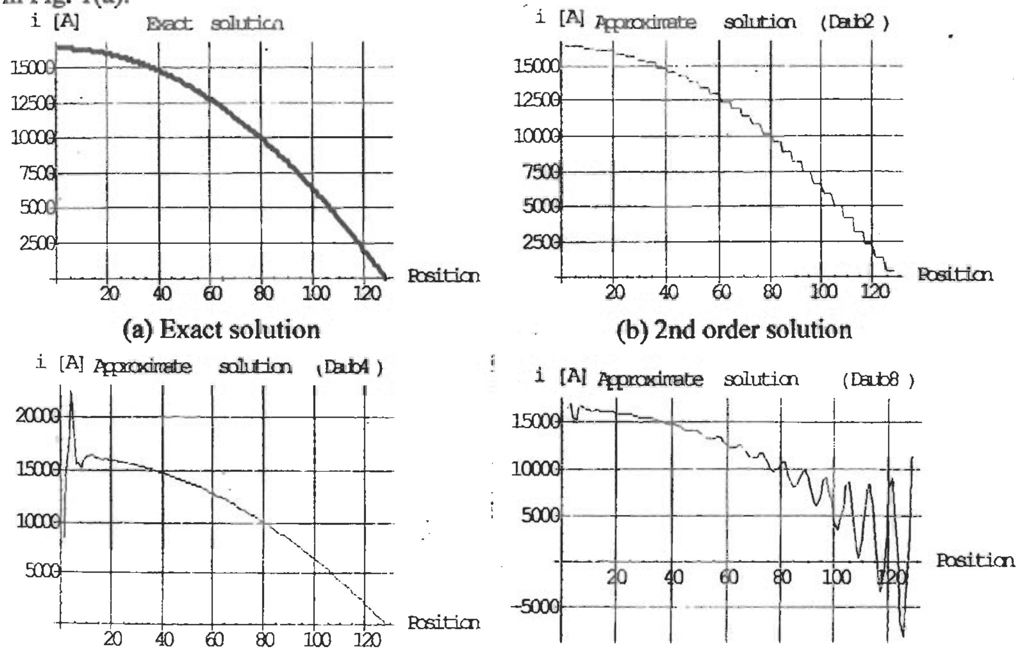
Figures 1(a) and 1(b) shows the example of $n=32$ by $m=128$ system matrix C and its wavelet spectrum C', respectively.



(a) *32* by *128* system matrix (b) Wavelet spectrum matrix

Fig. 1. Example of the system matrix and its wavelet spectrum

By taking the top *32* by *32* square region in Fig.1(b) into account for an approximate inverse matrix, it is possible to obtain an approximate solution vector **X** in Eq.(1). This is the basic wavelet solution strategy for the ill posed system of equations. Wavelet solution strategy using low order base function, e.g. 2nd order Daubechies or Haar, yields a stairway approximate solution shown in Fig. 1(b), while exact solution is shown in Fig. 1(a).



(a) Exact solution (b) 2nd order solution

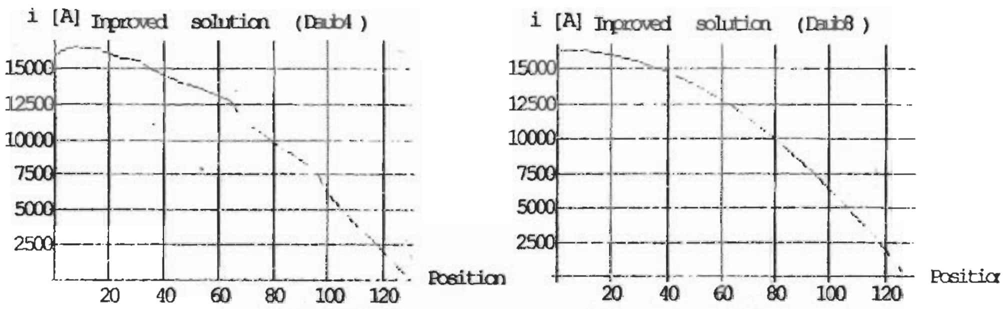
(c) 4th order solution (d) 8th order solution

Fig. 2. Typical wavelets solutions.

However, employment of higher order base functions, e.g. 4th , 8th order Daubechies, yields the solutions smoothed but containing large error at the edges of the solution vectors as shown in Figs. 1(c) and 1(d).

IMPROVED WAVELETS SOLUTION

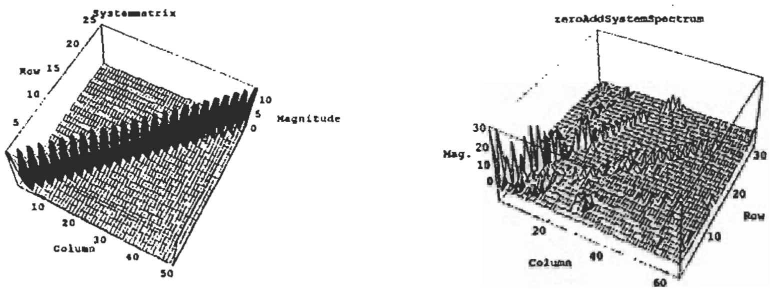
In order to clarify the error caused by the higher order base functions, we checked up a nature of data compression by the wavelets transform. As a result, it is clarified that the data compression process by using the higher order wavelets to a data having a discontinuity at the beginning or ending part always accompanies with a spiky error. Further, this spiky error has been caused by an assumption of periodical data structure[8]. One of the ways to change the data structure is to add the zero elements to the data vector. This is because the zero added data may be regarded as the periodic data having zero duration. The results are shown in Figs. 3(a) and 3(b) which correspond well to the exact solution in Fig.2(a).



(a)Daubechies 4^{th} order (b)Daubechies 8^{th} order

Fig.3. Improved solutions using the higher order base functions

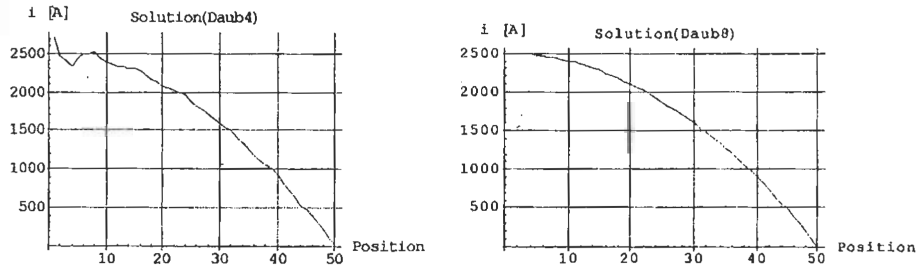
This zero addition method makes it possible to apply the wavelets strategy to the system of equations whose system matrix is not composed of the order not the power of 2, e.g. $n=25$ and $m=50$. Figures 4(a) and 4(b) show the system matrix ($n=25$, $m=50$) and wavelet spectrum of its zero added system matrix.



(a) *25* by *50* system matrix (b) Wavelet spectrum of a zero added system matrix

Fig. 4. Example of the system matrix and its wavelet spectrum

Figures 5(a) and 5(b) show the solutions using the Daubechies 4th and 8th order base functions, respectively.



(a) Daubechies 4th order (b)Daubechies 8th order

Fig. 5. Improved wavelet solutions(*25* by *50*)

To check up the nature of wavelet solution, we applied a multiresolutional analysis to the solution shown in Fig. 5(b). Figure 6 shows the results of multiresolutional analysis. The level in wavelet multiresolutional analysis simply corresponds to the space frequency. Thereby, Fig.6 suggests that the solution in Fig. 5(b) is composed of the low frequency components. In the other words, the wavelet solution neglects the higher frequency components contained in the exact solution.

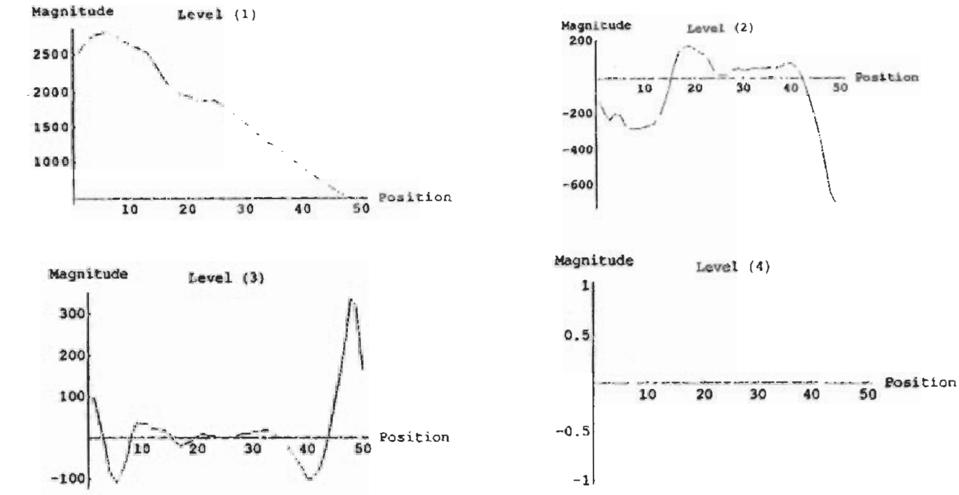
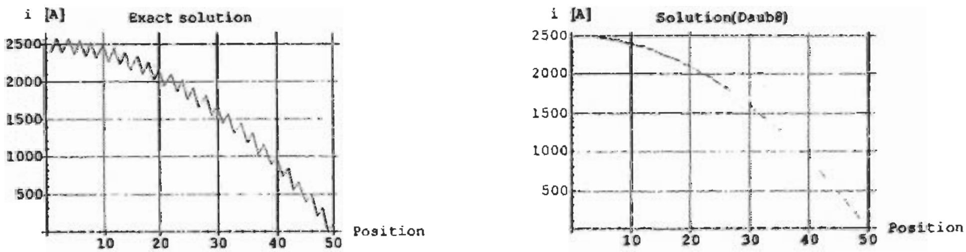


Fig. 6. Multiresolutional analysis of the solution in Fig. 5(b).

Figure 7(a) shows an exact solution containing a high frequency, while the wavelet solution shown in Fig. 7(b) does not include the high frequency components. Thus, it is obvious that the wavelet solution is one of the approximate solution neglecting the high frequency components in the exact solution.



(a) Exact solution (b) Wavelet solution

Fig.7. Comparison of the wavelet and exact solution containing high frequency component.

CONCLUSIONS

In this paper, we have proposed the wavelet strategy for the ill posed inverse system of equations. The two dimensional wavelet analysis has been applied to the rectangular system matrix regarded as an image data. And the approximate inverse matrix of the system has been obtained from a square part of the wavelet spectrum. Applying the inverse wavelet transform to the approximate inverse matrix in the wavelet spectrum space yields the approximate inverse matrix in the original space. The example concerning to the current estimation from the locally measured magnetic fields has demonstrated the validity of our approach. Further, we have proposed a methodology in order to improve the wavelets solutions obtained by means of image matrix. Key idea is to add the zero elements to the system matrix. This has made it possible to remove the noise caused by data compression. Thus, we have succeeded in obtaining the dramatically improved solutions of the ill posed system of equations.

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